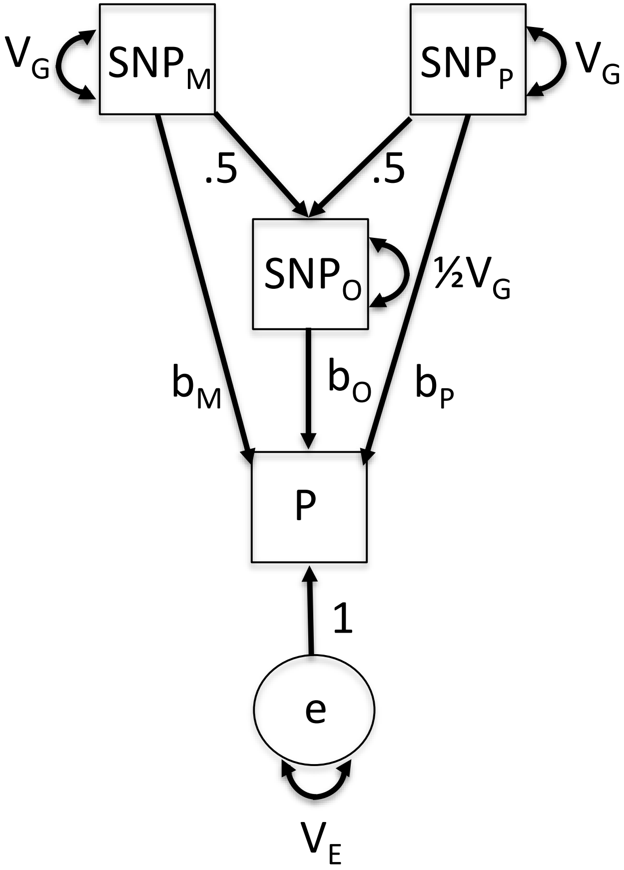
ANSWERS

Exercise: Deriving Expected Covariances



$COV(SNP_O, P) = b_O*V_G + ½*b_M*V_G + ½*b_P*V_G$

$VAR(P) = b_O^2*V_G + b_M^2*V_G + b_P^2*V_G + b_Ob_MV_G + b_Ob_PV_G + V_E$

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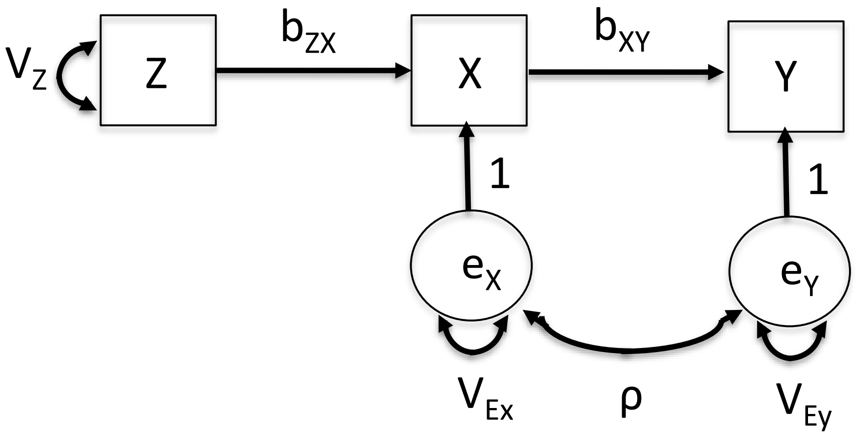
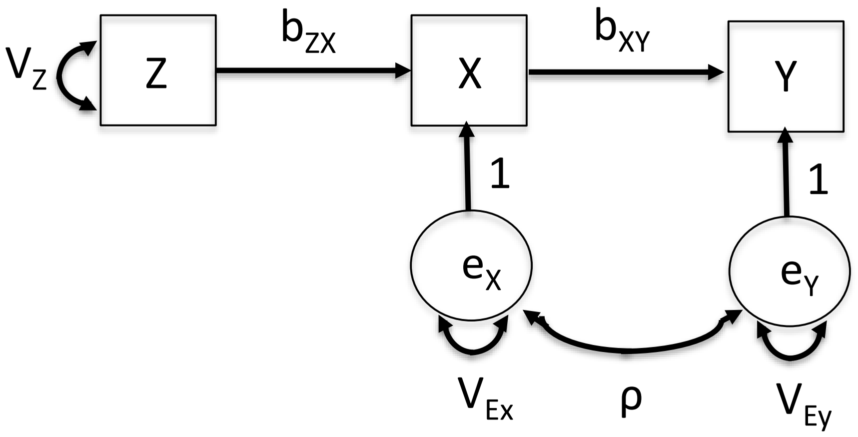
Therefore random measurement error in the X variable biases the regression coefficient towards the null, whereas random measurement error in Y does not bias the regression coefficient (although it will decrease power to detect an effect). This is known as “regression dilution bias”.

Mendelian Randomization



Mendelian Randomization



$$\boldsymbol{\Sigma(\theta)} = \begin{matrix} V_Z & b_{zx}V_z & b_{XY}b_{zx}V_z \\ b_{zx}V_z & b^2_{zx}V_z+V_{Ex} & b_{XY}(b^2_{zx}V_z+V_{Ex})+\rho \\ b_{XY}b_{zx}V_z & b_{XY}(b^2_{zx}V_z+V_{Ex})+\rho & b_{xy}{}^2(b^2_{zx}V_z+V_{Ex})+2b_{xy}\rho+V_{EY} \end{matrix}$$

Likelihood Ratio Test

The likelihood under the null hypothesis of a fair coin is:

(0.5)^65*(0.5)^35

The likelihood under the alternative hypothesis at the MLE is:

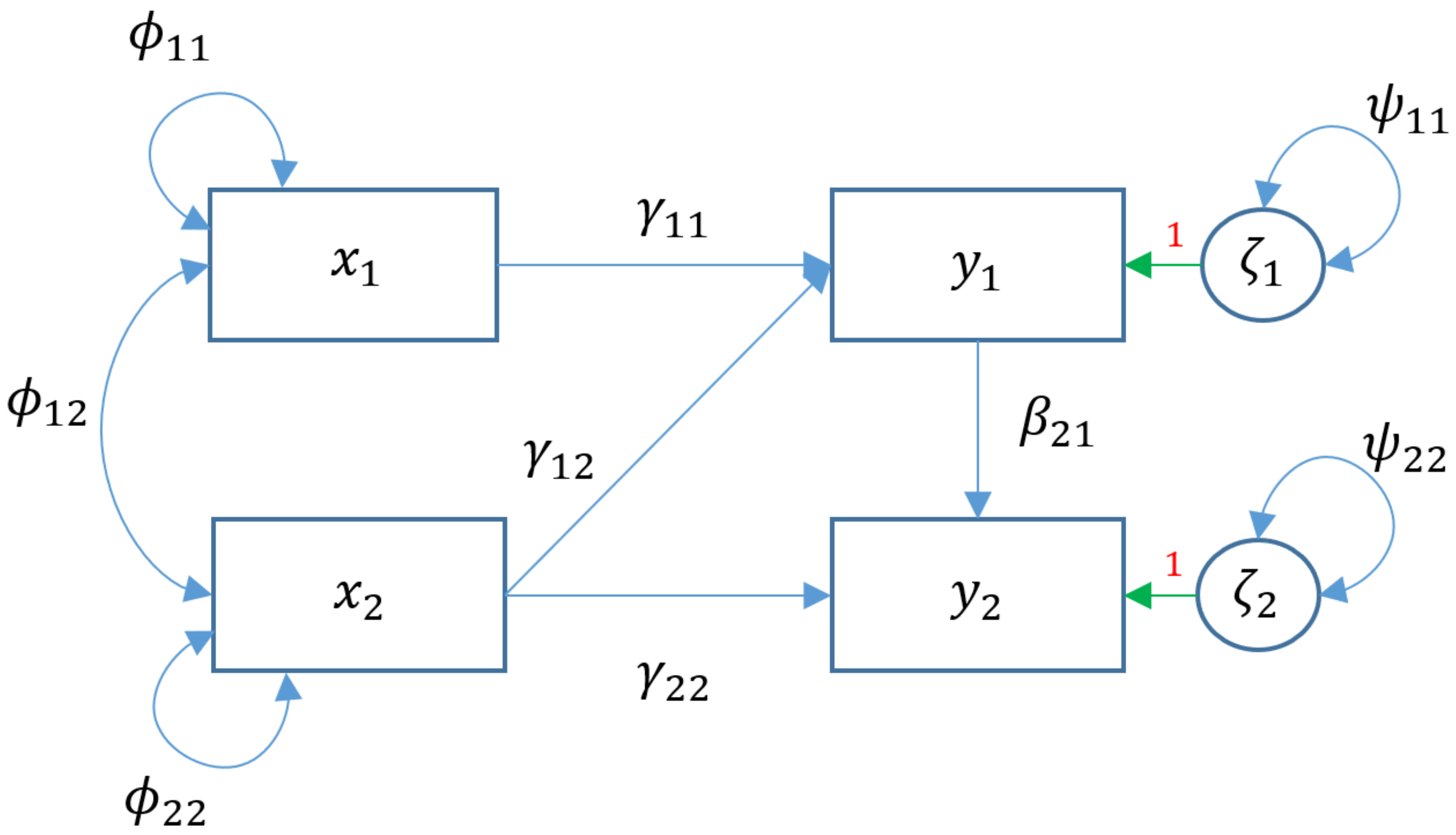
(0.65)^65*(0.35)^35

Twice the difference in log-likelihood is distributed as a chi-square with one degree of freedom:

```
chi <- -2*log((0.5)^65*(0.5)^35) - -2*log((0.65)^65*(0.35)^35)
pchisq(q = chi, df=1, ncp = 0, lower.tail = FALSE, log.p = FALSE)
p = 0.002500663
```

The coin is not fair

Exercise: Deriving Expected Covariances

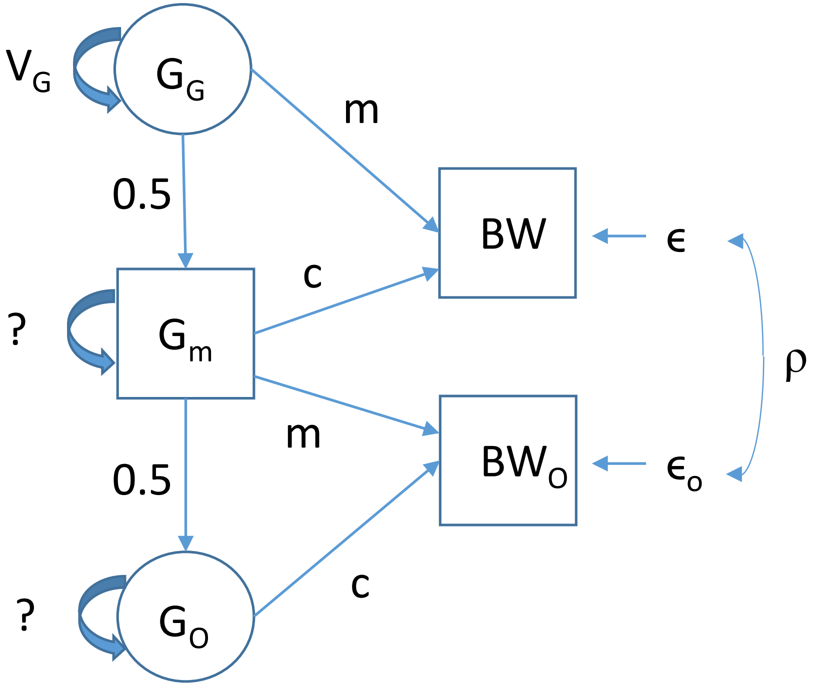


$\mathrm{COV}(x_1, \zeta_1) = 0$

$\mathrm{COV}(x_1, y_2) = \beta_{21}\gamma_{11}\phi_{11}$
$+ \beta_{21}\gamma_{12}\phi_{12} + \gamma_{22}\phi_{12}$

$\mathrm{VAR}(y_2) = \beta^2{}_{21}\psi_{11} + \beta^2{}_{21}\gamma^2{}_{11}\phi_{11}$
$+ \beta^2{}_{21}\gamma^2{}_{12}\phi_{22} + 2\beta_{21}\gamma_{12}\phi_{22}\gamma_{22} +$
$2\beta_{21}\gamma_{11}\phi_{12}\gamma_{22} + 2\beta_{21}\gamma_{11}\phi_{12}\gamma_{12}\beta_{21}$
$+\gamma^2{}_{22}\phi_{22} + \psi_{22}$

Exercise



(1) Assuming that the genotypic variance doesn't change across generations what is the variance of the G_m and G_O terms? $0.75V_G$

(2) Derive cov(BW, BW_O) = $0.5m^2V_G + 0.25mcV_G + cmV_G + 0.5c^2V_G + \rho$