

Estimation & interpretation of genetic variance

UQ Winter School, June 2022

Dr Kathryn Kemper

(with thanks to Loic, JZ and Jian Yang)



Pruned and modified version of 5-part workshop given by Dr Loic Yengo@ISGW

“Heritability of individual level data”

e.g. <https://www.youtube.com/watch?v=Cjn5AtNPjzE>

Outline

- Definition of heritability
- Estimation
 - Relationship matrices
 - HE regression
 - REML
- Interpreting h^2 estimates

Definition

Heritability (h^2):

quantifies the degree to which inter-individual differences and resemblance in the population are due to genetic factors.



Chial, H. (2008) Polygenic inheritance and gene mapping.
Nature Education 1(1):17

Definition

The value of the trait, or phenotype (P), can be modelled as

$$P = A + E$$

(additive) Non-genetic
genetic factors factors

$$\text{then } h^2 = \frac{\sigma_A^2}{\sigma_P^2}$$

the heritability is the proportion of phenotypic variance (σ_P^2) attributable to additive genetic effects (σ_A^2)

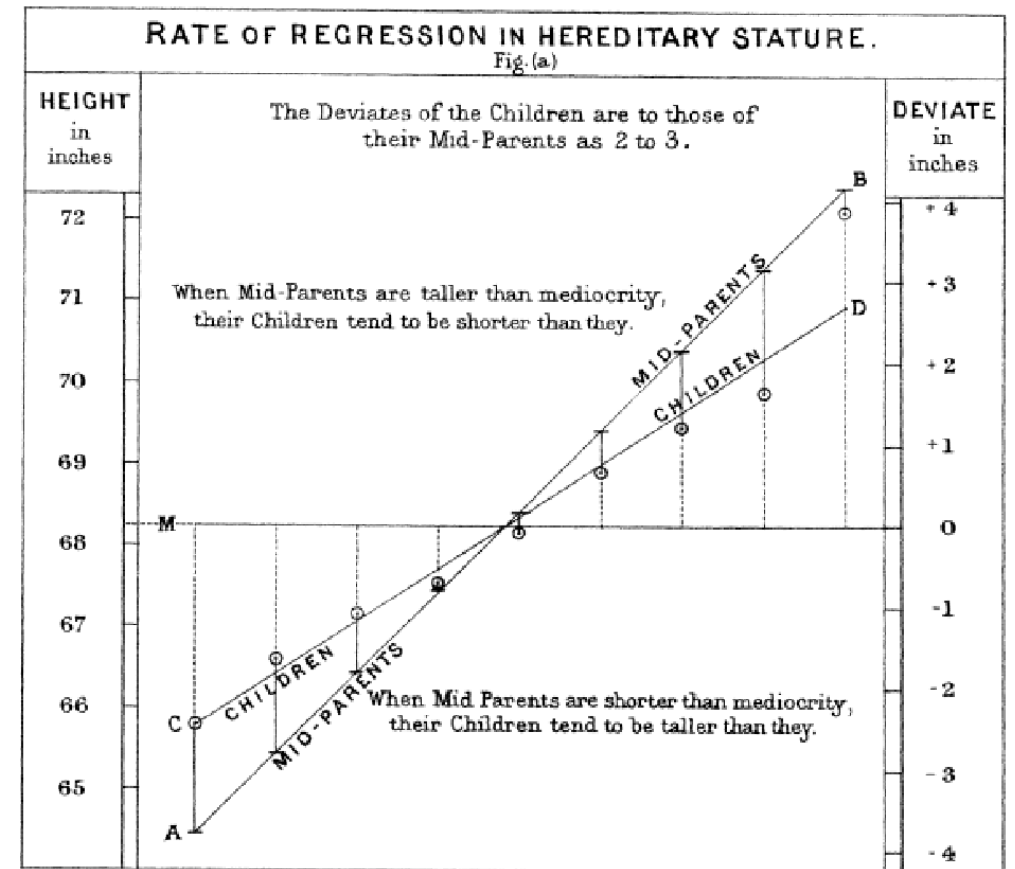
Heritability ranges between 0 and 1



Chial, H. (2008) Polygenic inheritance and gene mapping. Nature Education 1(1):17

Definition

- How can we estimate 'A' when we can't observe the true genetic effects?



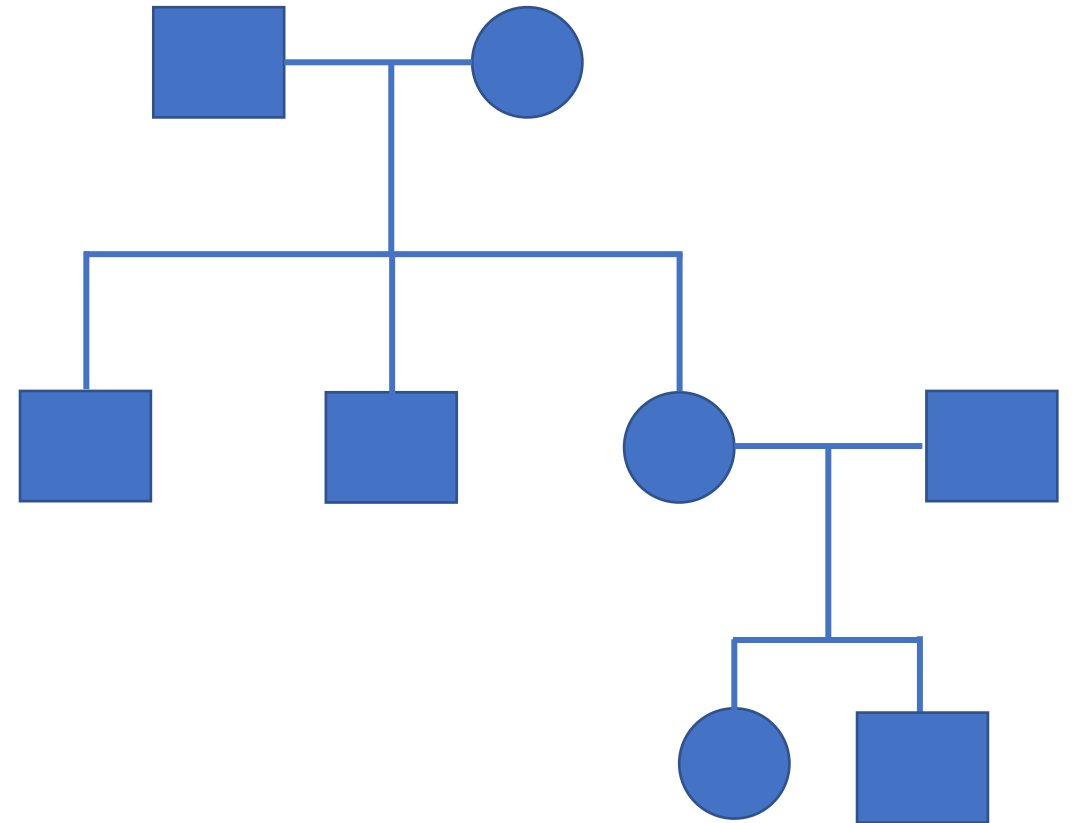
Galton (1886)

What are 'average' relationships?

- Animal and plant breeders have used 'average' or pedigree relationships since 1950's to drive genetic change
- Human geneticists have mostly relied on comparing MZ and DZ twins
- These approaches rely on the average genetic relationship between relatives

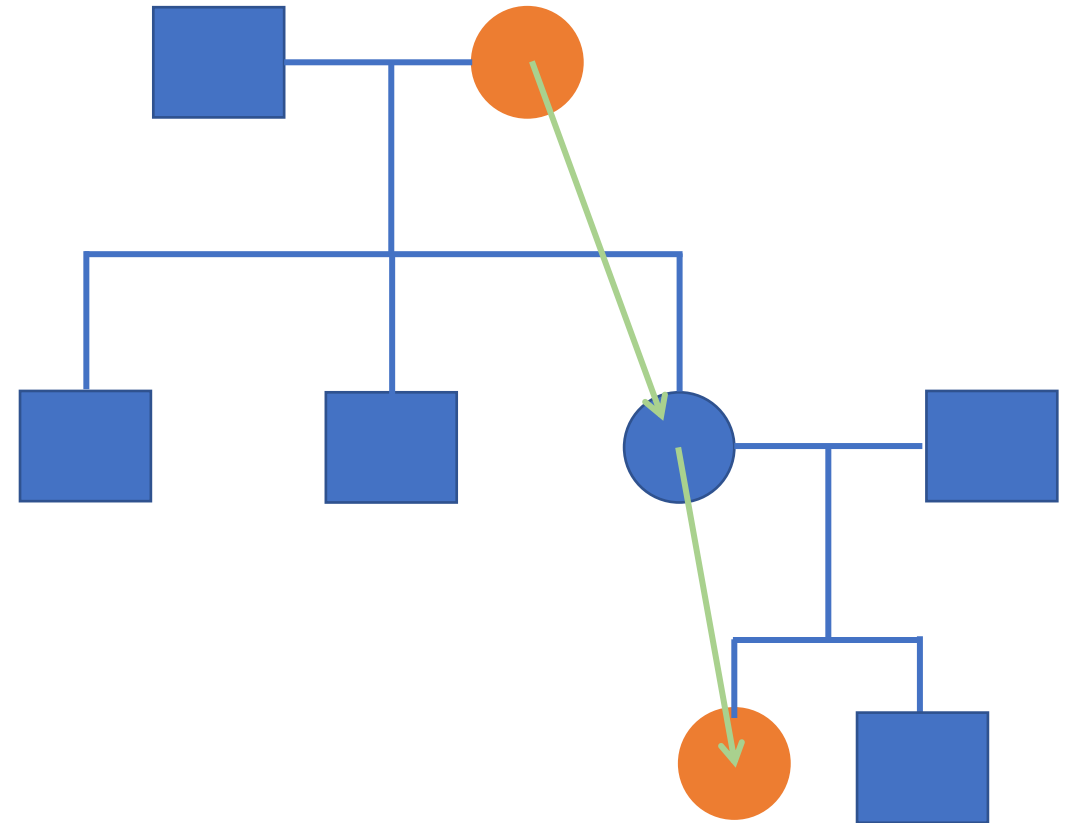
What are 'average' relationships?

Relationship	Relationship co-efficient (π)
MZ twins	$1.0 = 0.5^0$
Parent-child	$0.5 = 0.5^1$
Full-sibs/DZ twins	$0.5 = 2 \times 0.5^2$
Half-sibs	$0.25 = 0.5^2$
Avuncular	$0.25 = 2 \times 0.5^3$
Grandparent-child	$0.25 = 0.5^2$
1 st cousins	$0.125 = 2 \times 0.5^4$



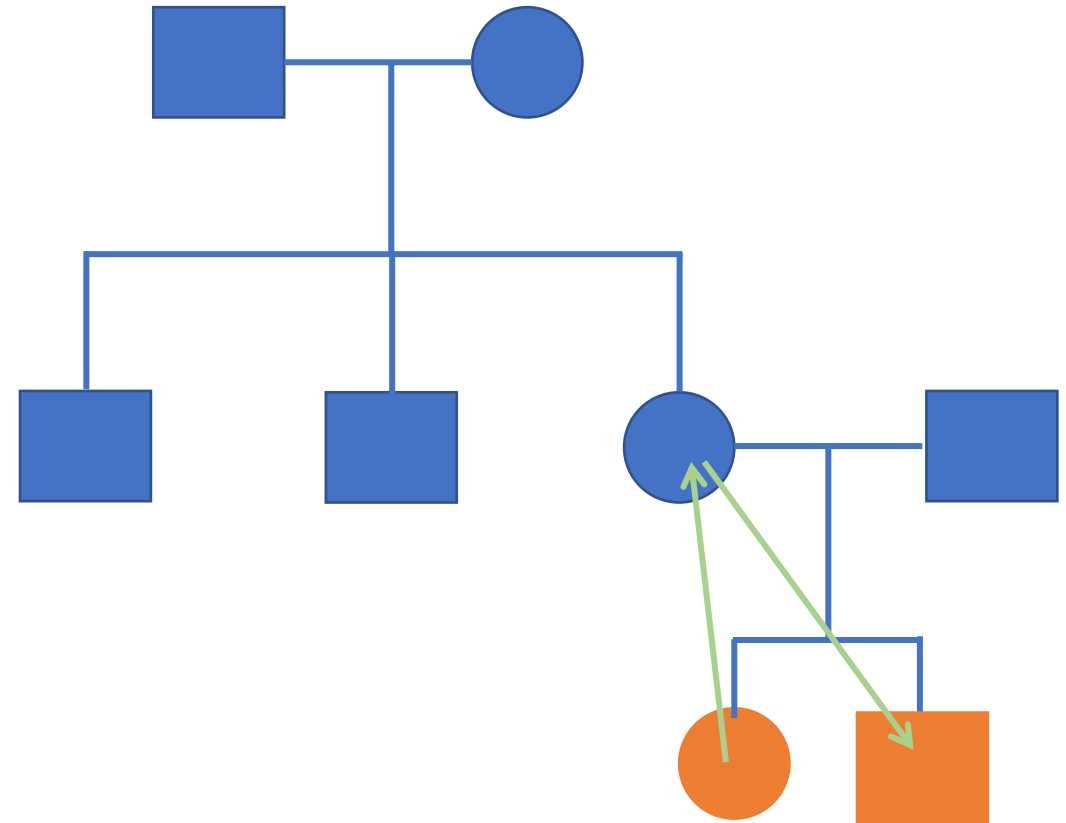
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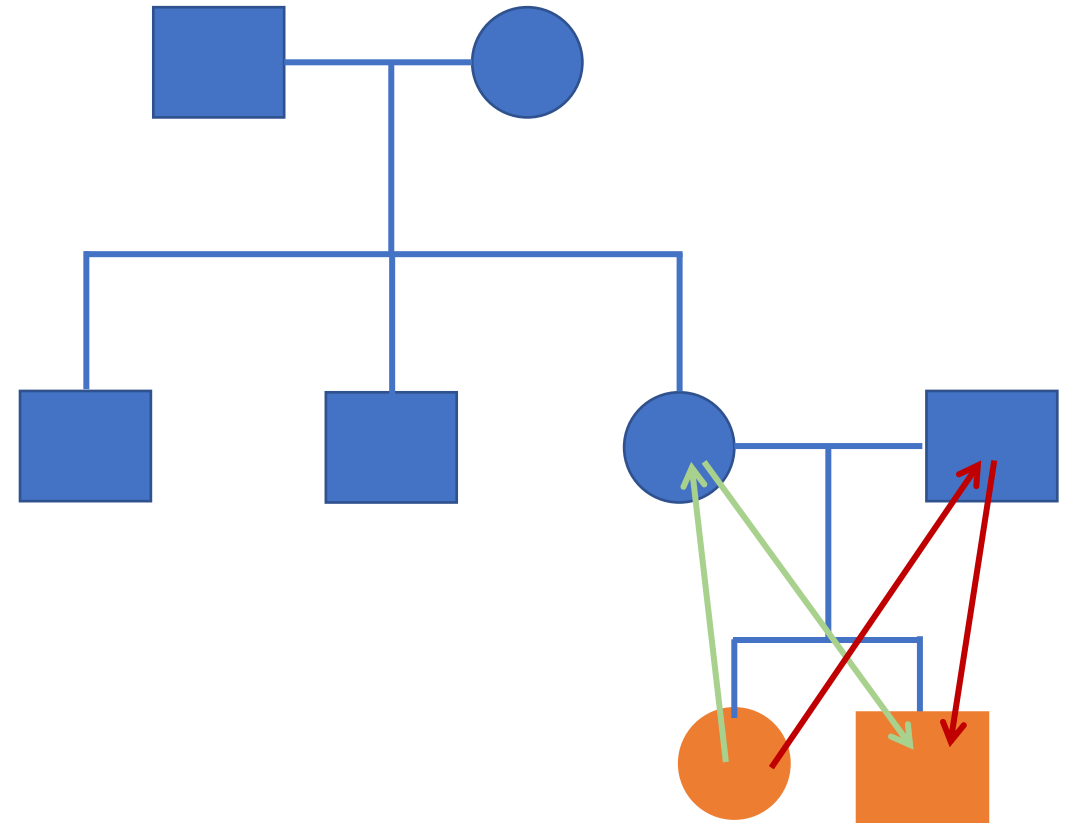
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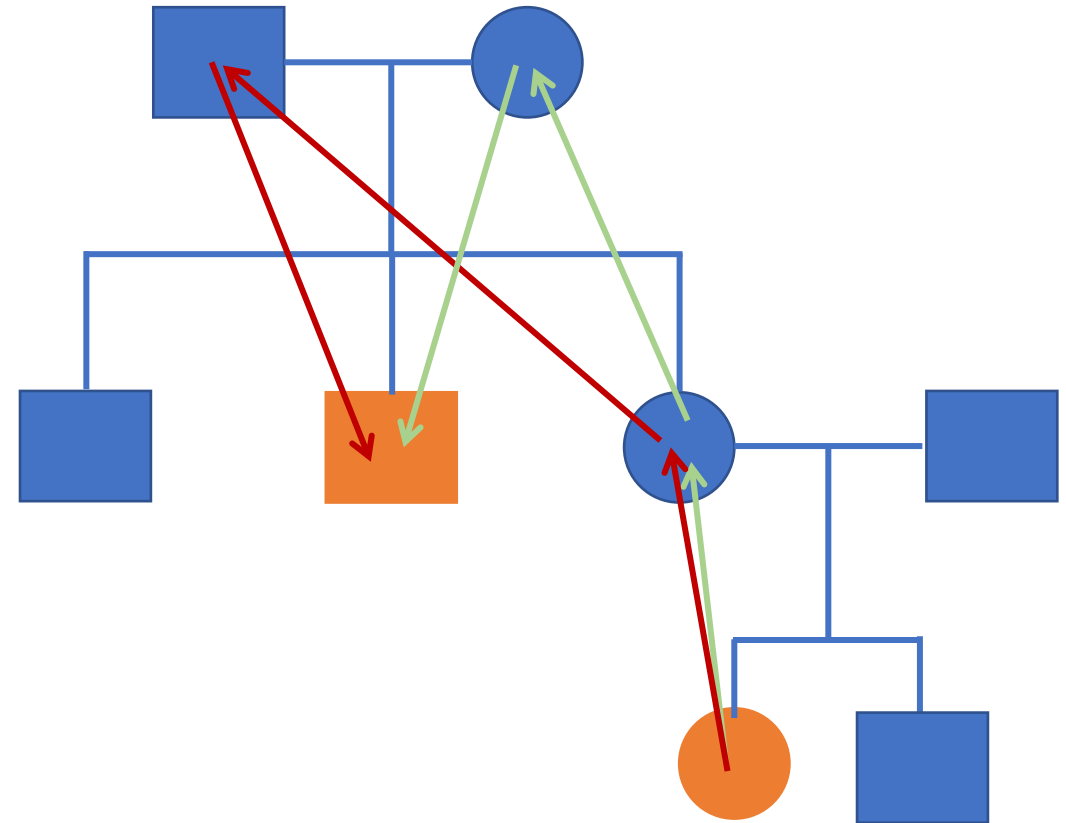
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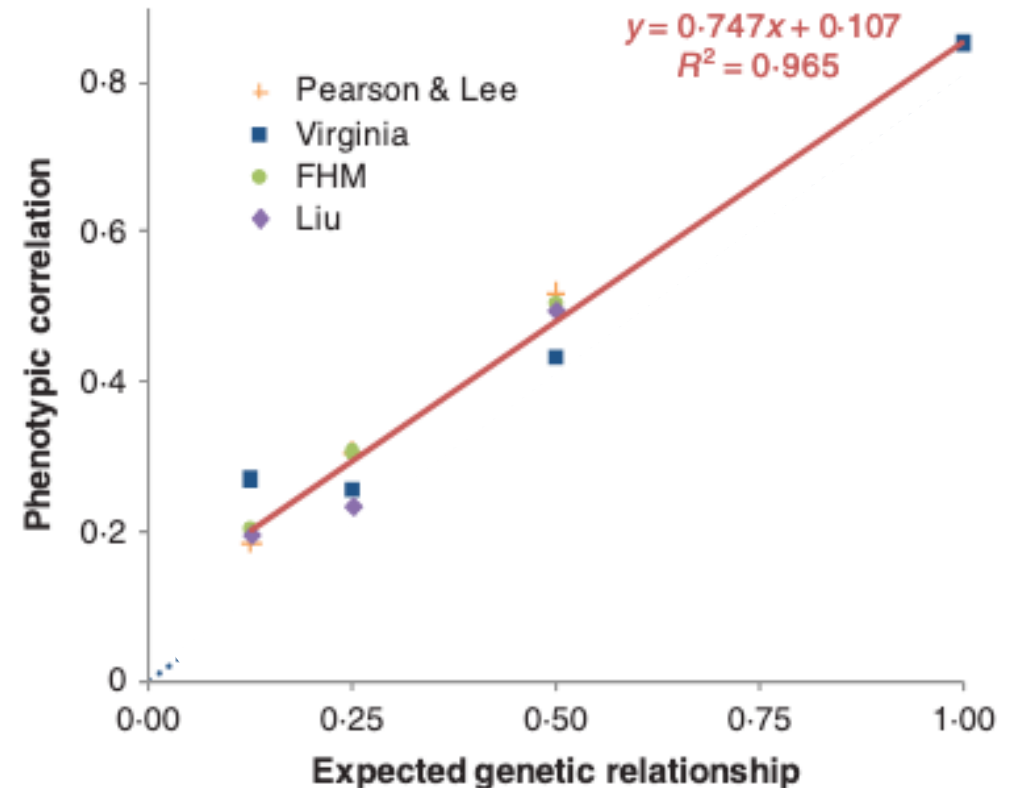
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Definition

We can estimate the heritability of a trait using average relationships, e.g.

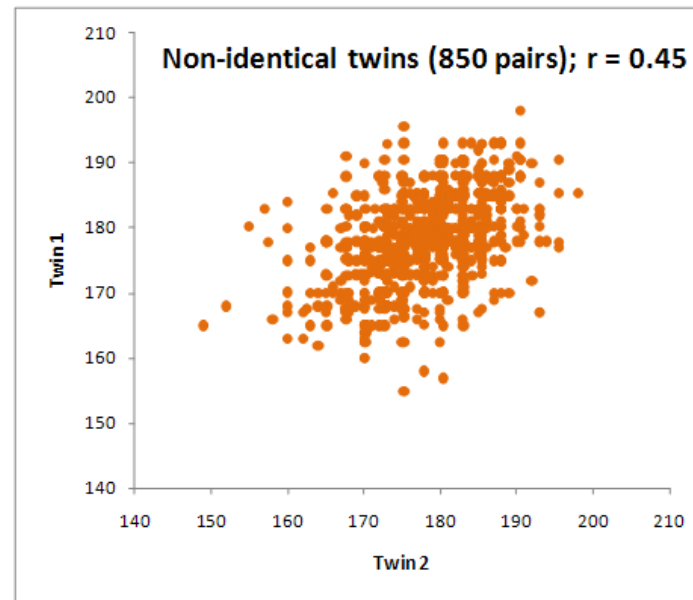
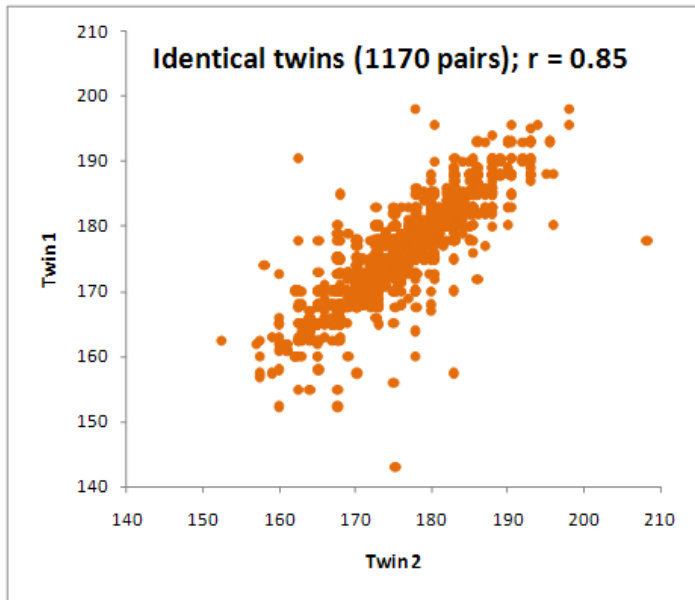
$$\text{corr}(Y_i, Y_j) = h^2 \pi_{ij} + \text{residual}$$



Visscher, McEvoy & Yang (2010) *Genet. Res.* 92:371-379.

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Twin-based estimate heritability:

$$r(\text{MZ}) = [\text{var}(\text{G}) + \text{var}(\text{E})] / \text{var}(\text{P})$$

$$r(\text{DZ}) = [0.5 \text{var}(\text{G}) + \text{var}(\text{E})] / \text{var}(\text{P})$$

$$2 [r(\text{MZ}) - r(\text{DZ})] = 2 [0.5 \cdot \text{var}(\text{G})] / \text{var}(\text{P}) = h^2$$

Why all the fuss about h^2 ?

- 1) The heritability of a trait gives an upper bound for the accuracy of genetic predictors of that trait.
- 2) The heritability predicts the response to (natural/artificial) selection.
- 3) The heritability predicts an individual's risk to develop a certain disease knowing they have affected relatives.
- 4) The heritability influences the statistical power of genome-wide association studies (GWAS)

Misconceptions about heritability

Heritability is an estimate, based on many assumptions. Beware.

Heritability is a property of a trait, in a population, at a given time. It is not fixed.

A low heritability does not necessarily mean the trait is not genetically determined.

- It suggests that non-genetic factors account for more variation.
- Is there phenotypic variation? (e.g. number of fingers)

Methods to estimate heritability

$$\text{corr}(Y_i, Y_j) = h^2 \boldsymbol{\pi}_{ij} + \text{residual}$$

Covariance between 'relatives' is fundamental to h^2 estimation.

Methods differ in the approaches to combine $\text{corr}(Y_i, Y_j)$ and $\boldsymbol{\pi}_{ij}$, e.g.

- we can estimate relationships using pedigrees or genetic markers
- we can use a regression, ANOVA or REML framework for parameter estimates

Relationship matrices

A relationship matrix (of dimension $n \times n$, where n = number of individuals) is a square symmetrical matrix where each element defines the relationship between two individuals.

e.g. for pedigree relationships $A_{ij} = 0.5$ for full-sibs i and j

Relationship matrices – average relationship

A relationship matrix (of dimension $n \times n$, where n = number of individuals) is a square symmetrical matrix where each element defines the relationship between two individuals.

e.g.

ID	Mother	Father
1	-	-
2	-	-
3	1	2
4	1	2
5	1	2
6	-	-
7	5	6

Lower triangle A matrix:

1.0						
0	1.0					
0.5	0.5	1.0				
0.5	0.5	0.5	1.0			
0.5	0.5	0.5	0.5	1.0		
0	0	0	0	0	1.0	
0.25	0.25	0.25	0.25	0.5	0.5	1.0

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There are many ways to calculate a relationship matrices when using SNP data. We will focus on a standard estimator implemented in the software **GCTA**.

Standard GRM estimator

$$\hat{\pi}_{jk} = \frac{1}{m} \sum_i \frac{(x_{ij} - 2p_i)(x_{ik} - 2p_i)}{2p_i(1 - p_i)}$$

where, x_{ij} and x_{ik} are the minor allele count ($x_{ij}, x_{ik} = 0, 1$ or 2) at SNP i for individuals j and k respectively, p_i the minor allele frequency (MAF) of SNP i and m the number of SNPs used to calculate the GRM.

Example of GRM between $N=3$ individuals
(over $m=1000$ SNPs)

```
[$bash] zless myGRM.grm.gz
```

```
1 1 1000 0.99
```

```
1 2 1000 -0.01
```

```
1 3 1000 0.01
```

```
2 2 1000 1.03
```

```
2 3 1000 0.03
```

```
3 3 1000 1.01
```

ANALYSIS

nature
genetics

Common SNPs explain a large proportion of the heritability for human height

Jian Yang¹, Beben Benyamin¹, Brian P McEvoy¹, Scott Gordon¹, Anjali K Henders¹, Dale R Nyholt¹, Pamela A Madden², Andrew C Heath², Nicholas G Martin¹, Grant W Montgomery¹, Michael E Goddard³ & Peter M Visscher¹

SNPs discovered by genome-wide association studies (GWASs) account for only a small fraction of the genetic variation of

of variation that their effects do not reach stringent significance thresholds and/or the causal variants are not in complete linkage

Estimating genetic variance

- Haseman-Elston (HE) regression, 'method of moments'

$$\text{corr}(Y_i, Y_j) = h^2 \pi_{ij} + \text{residual}$$

- ANOVA for balanced designs
- Restricted Maximum Likelihood (REML)

Haseman-Elston (HE) regression

HE regression estimates h^2 by regressing $Z_j Z_k$ onto $\hat{\pi}_{jk}$,

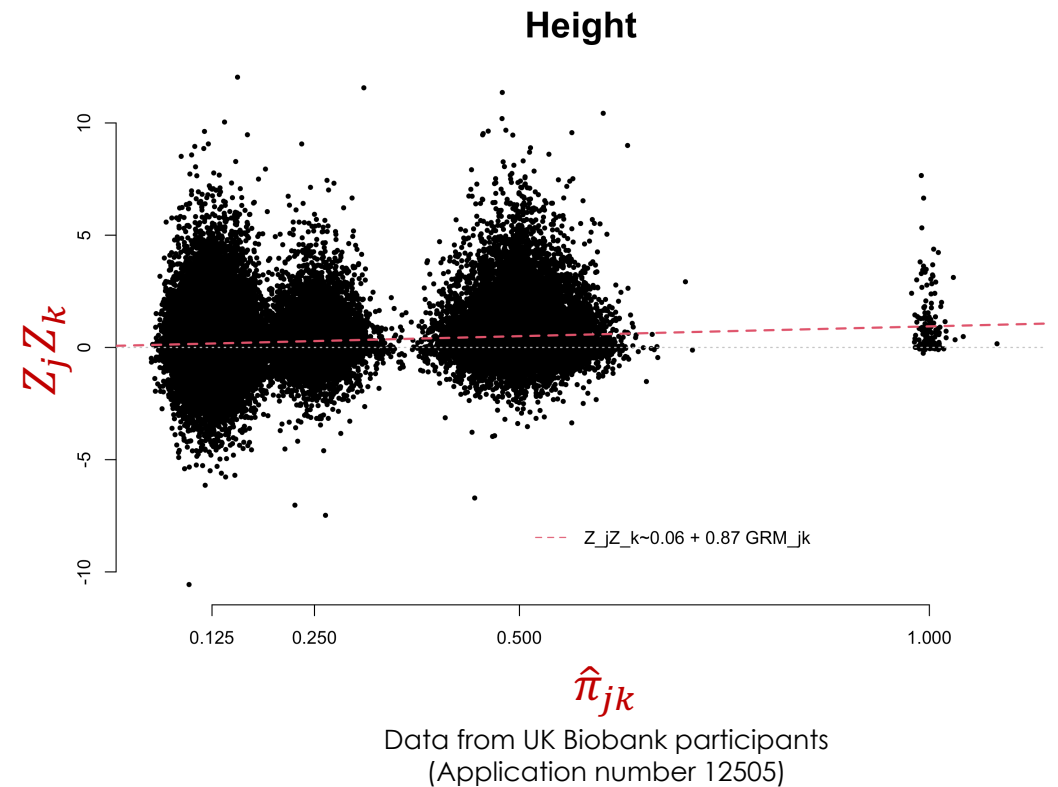
where

$Z_j = (Y_j - \text{mean}(Y))/\text{sd}(Y)$, and

$Z_k = (Y_k - \text{mean}(Y))/\text{sd}(Y)$

Thus,

$$E[Z_j Z_k] = \text{corr}(Y_j, Y_k)$$



$$E[Z_j Z_k | \hat{\pi}_{jk}] = 0.06 + 0.87 \hat{\pi}_{jk} \Rightarrow \hat{h}_{HE}^2 \sim 0.87.$$

HE regression with GCTA

Step 1: Calculate the GRM

```
gcta64 --bfile myDataInPLINKformat --make-grm-bin --out myData
```

HE-CP					
Coefficient	Estimate	SE_OLS	SE_Jackknife	P_OLS	P_Jackknife
Intercept	-9.89933e-05	0.000235661	6.36354e-06	0.674437	1.44216e-54
V(G)/Vp	0.405919	0.0182643	0.0352467	1.99052e-109	1.0898e-30
HE-SD					
Coefficient	Estimate	SE_OLS	SE_Jackknife	P_OLS	P_Jackknife
Intercept	-0.999932	0.00033015	0.0179081	0	0
V(G)/Vp	0.40622	0.0255874	0.0371021	9.335e-57	6.74268e-28

Step 3: Run GCTA to estimate heritability of trait 1 using HE regression

```
gcta64 --grm myData --pheno phenotype.txt --mpheno 1 --HEreg --out myHE_estimates
```

[generates 2 files: myHE_estimates.log, myHE_estimates.HEreg]

REML estimation

- Mixed model: $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{Z}\mathbf{u} + \mathbf{e}$,
 - where $\mathbf{u} \sim \mathbf{N}(\mathbf{0}, \mathbf{A}\sigma_a^2)$ & $\mathbf{e} \sim \mathbf{N}(\mathbf{0}, I_n\sigma_e^2)$
- Use REstricted Maximum Likelihood to estimate parameters
 - ‘restricted’ in that we also have fixed effects i.e. $\mathbf{y} \sim N(\mathbf{X}\boldsymbol{\beta}, \sigma_a^2\mathbf{A} + \sigma_e^2I_n)$
 - REML accounts for df lost due to estimation of fixed effects
- Specifically in human genetics, often called GREML (Genome-based REML)
- Genome-based :-
 - using SNPs to determine relationships
 - assume that genetic effects are a linear combination of SNP effects
 - “big-p little-n” problem (where n = samples & p = predictors)

GREML estimation with GCTA

Run GCTA to estimate heritability of trait 1 using GREML

```
gcta64 --grm myData --pheno phenotype.txt --mphenos 1 --reml --out myGREML_estimates
```

[generates 2 files: myGREML_estimates.log, myGREML_estimates.hsq]

Source	Variance	SE
V(G)	0.398550	0.023990
V(e)	0.578277	0.019175
Vp	0.976827	0.019107
V(G)/Vp	0.408004	0.020539
logL	-2722.000	
logL0	-2932.909	
LRT	421.817	
df	1	
Pval	0.0000e+00	
n	6000	

The significance of h^2_{SNP} is assessed by likelihood ratio test (LRT)

$$H_0: h^2_{\text{SNP}} = 0$$

$$H_1: h^2_{\text{SNP}} \neq 0$$

LRT = $2[L(H_1) - L(H_0)]$ is distributed as a half probability of 0 and a half probability of chi-squared with 1 d.f.

Interpreting h^2 estimates

- h^2 estimates are based on many assumptions, depending on your approach
 - e.g. h^2 -SNP depends on the SNP you use!
 - most models assume random mating
- Bias from shared environment
 - can be model explicitly, e.g. twin analysis (still biased?)
 - use only unrelated individuals ($\pi_{ij} < 0.05$)
 - within-family estimates
- G-E confounding – population stratification or parental effects
 - Fit PCs as covariates
 - Within-family analyses

Missing heritability



The case of the missing heritability

Manolio et al. 2009 Nature

$$h_{\text{GWAS}}^2 \leq h_{\text{SNP}}^2 \leq h_{\text{PED}}^2$$

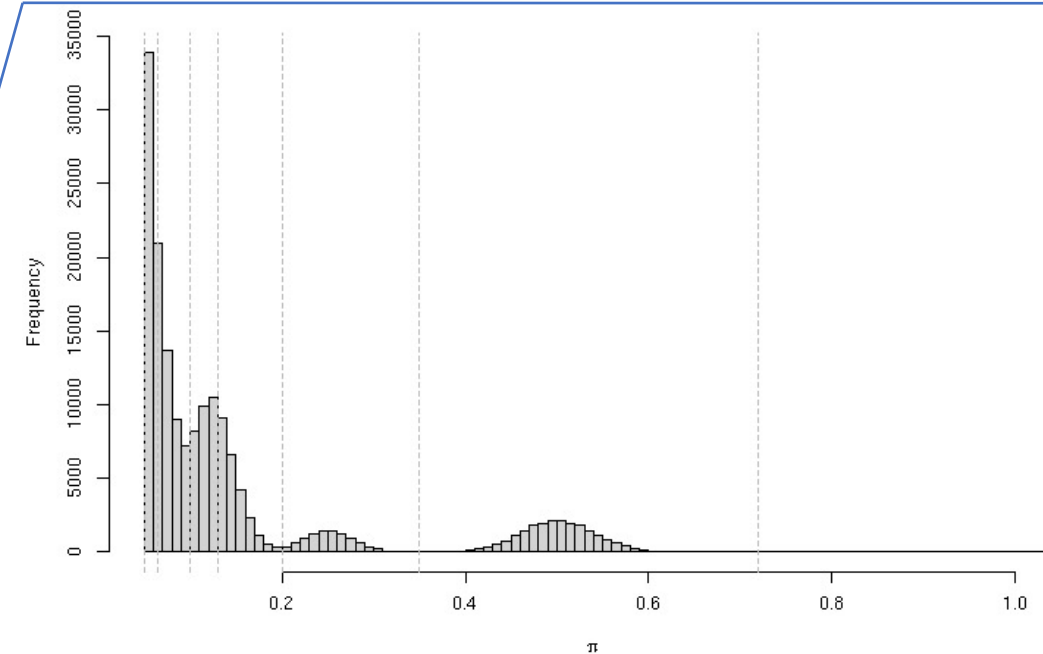
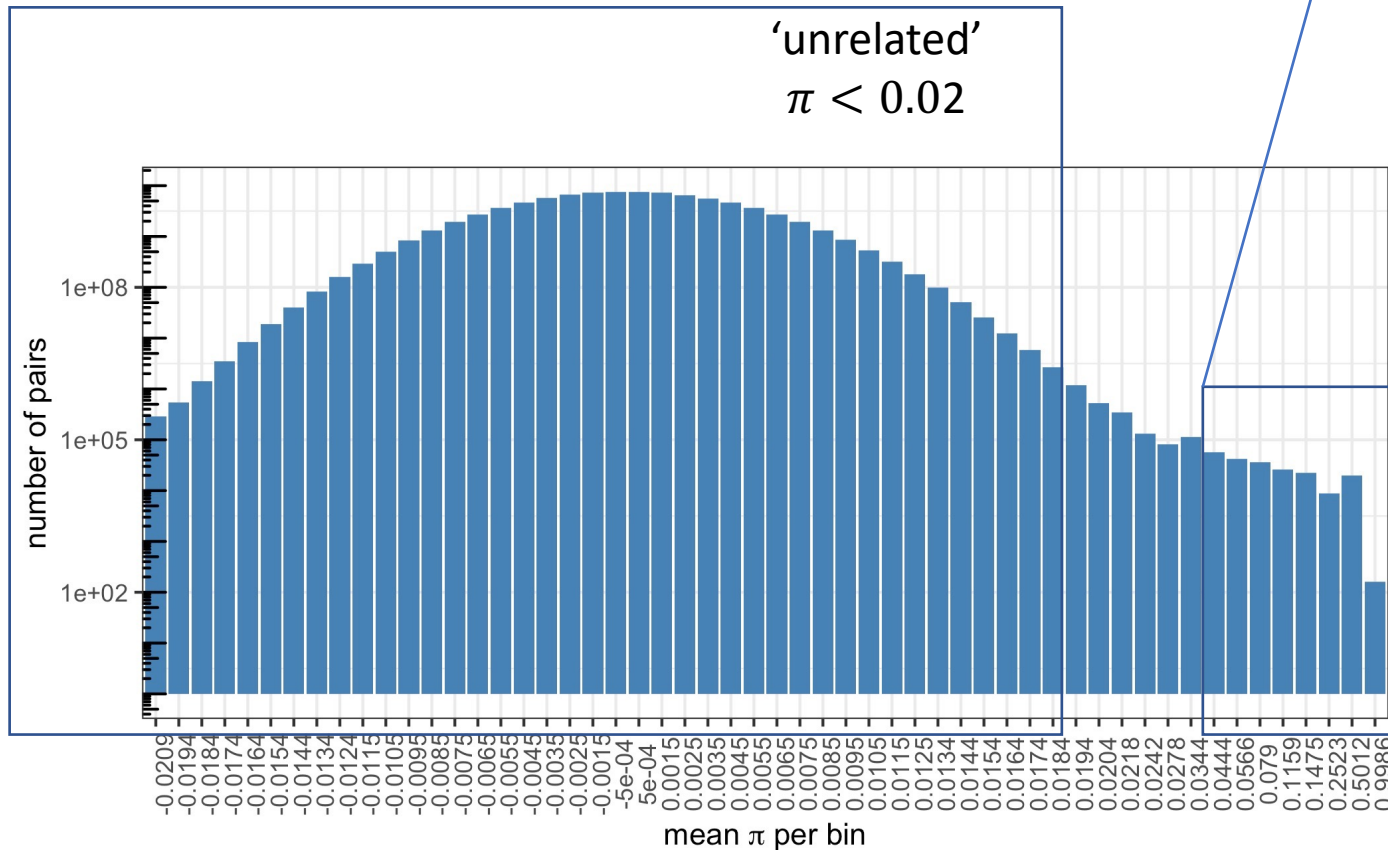
$h_{\text{PED}}^2 - h_{\text{GWAS}}^2$ is often denoted the “missing” heritability (e.g., 5% vs 80%).

$h_{\text{SNP}}^2 - h_{\text{GWAS}}^2$ is often denoted the “hidden/hiding” heritability.

$h_{\text{PED}}^2 - h_{\text{SNP}}^2$ is denoted the (still) missing heritability.

Genomic relationship matrix

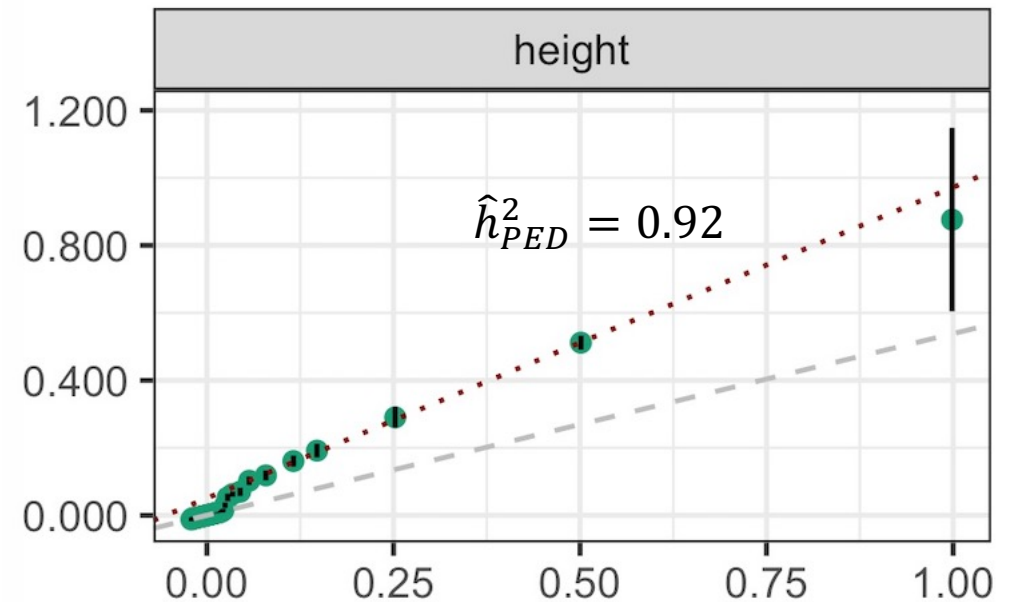
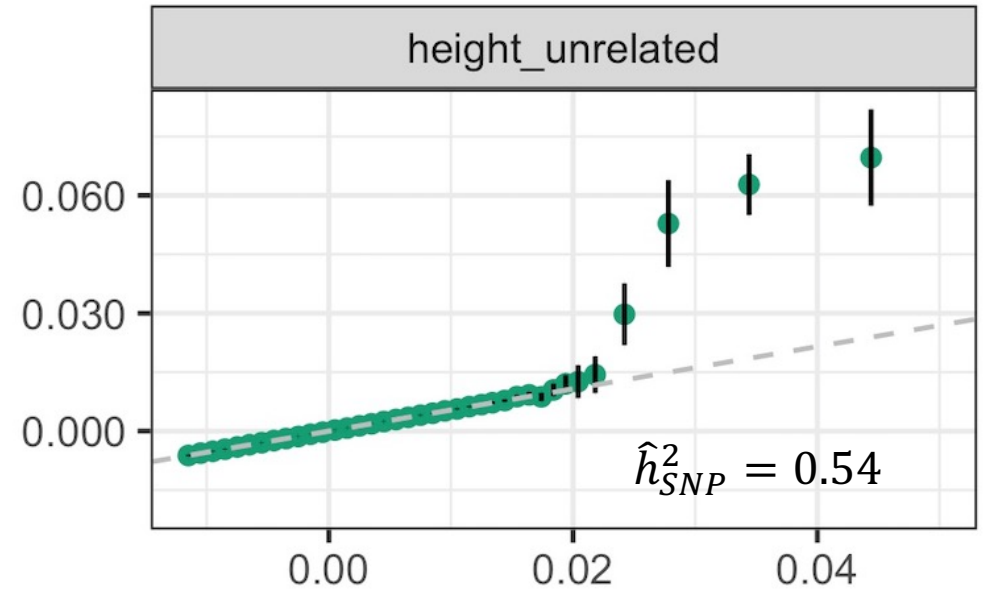
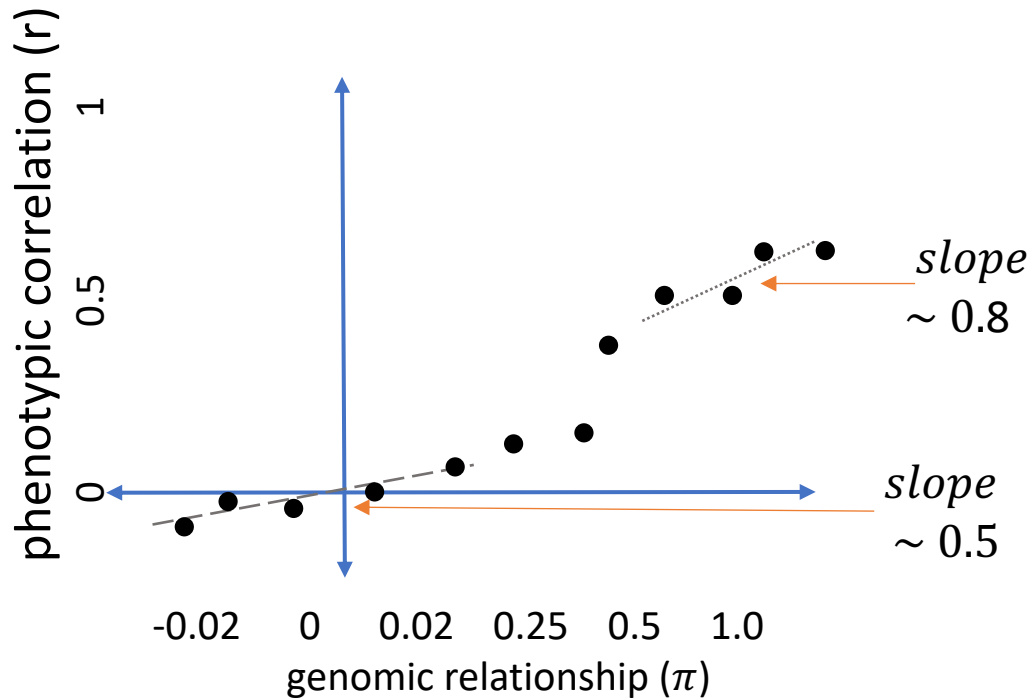
- 1.1M HapMap3 SNPs with MAF > 0.01 for 450K individuals
- constructed using GCTA (Yang et al. 2010)
- allocated pairs into 54 relationship 'bins' or groups



'close relatives'
 $\pi > 0.05$

Missing heritability? – human height

$$\text{corr}(Y_i, Y_j) = h^2 \pi_{ij} + \text{residual}$$



Practical – estimation of h^2 using GCTA

We will loosely follow a practical developed for STAT 3306-7306

- grm is located in /data/module4/prac8/QIMRX_no_twin.grm.gz
- Made with the following code in GCTA, e.g.
 - `gcta --bfile <plinkSNPFile> --make-grm.gz --out QIMRX_no_twin`
- Look at the 'gzip' version of the grm using '`zcat <file>.grm.gz | head`' at the command line

```
[ec2-user@analysis1 uqkkempe]$ zcat /data/module4/prac8/QIMRX_no_twin.grm.gz | head
1      1      2.658050e+05    9.824743e-01
2      1      2.657710e+05    4.307623e-01
2      2      2.657910e+05    9.971558e-01
3      1      2.615040e+05    1.788882e-03
3      2      2.614920e+05    1.014439e-03
3      3      2.615290e+05    1.000038e+00
4      1      2.657180e+05   -1.286868e-03
4      2      2.657040e+05   -2.567511e-04
4      3      2.614420e+05   -3.715489e-03
4      4      2.657430e+05    1.001568e+00
```

Practical – estimation of h^2 using GCTA

- Use GCTA to estimate the SNP-heritability for height, e.g.

```
gcta --grm /data/module4/prac8/QIMRX_no_twin \  
--pheno /data/module4/prac8/HT_T_X.pheno \  
--mpheno 1 --reml --out QIMRX_1
```

- View the resulting file at the command line using ‘more’

```
[ec2-user@analysis1 uqkkempe]$ more QIMRX_1.hsq  
Source  Variance      SE  
V(G)    0.637715      0.110157  
V(e)    0.384206      0.104987  
Vp      1.021921      0.027815  
V(G)/Vp 0.624036      0.103832  
logL    -1400.300  
logL0   -1418.770  
LRT     36.939  
df      1  
Pval    6.0953e-10  
n       2768
```

Practical – closer look @ the GRM

- Open R & use
 - `x = read.table("<file>.grm.gz")` to load the data into R
 - lower triangle GRM; columns are row #, column #, # SNPs, relationship value
- Make a true/false vector if the relationship value is a diagonal
 - `diagElement = x[,1] == x[,2]`
 - `sum(diagElement) ; head(diagElement)`
- Plot results
 - `hist(x[diagElement,4], breaks=2500, xlab="GRM diagonals")`
 - `hist(x[!diagElement,4], breaks=2500, xlab="GRM off-diagonals")`

Practical – removing relatives

- In the off-diagonal plot there were some large relationships
- In R use “`sum(x[!diagElement,4]>0.05)`” to find out how many pairs
- Now we are going to use GCTA to remove one member of the close relative pairs
 - `gcta --grm /data/module4/prac8/QIMRX_no_twin \
--grm-cutoff 0.05 --make-grm --out QIMRX_nr`
- Re-run SNP-heritability estimate with your new pruned matrix
 - `gcta --grm QIMRX_nr --pheno /data/module4/prac8/HT_T_X.pheno \
--mpheno 1 --reml --out QIMRX_nr_1`

Practical – Haseman-Elston regression

- He regression can be run in GCTA using --Hereg, e.g.
 - `gcta --grm QIMRX_nr --pheno /data/module4/prac8/HT_T_X.pheno --mpheno 1 \`
`--HEreg --out QIMRX_nr_1b`
- However today we're going to do it by-hand, in R!
- Open R:
 - `Hereg = read.table("/data/module4/prac8/he.grm.txt")`
 - `names(Hereg) <- c("ID1", "ID2", "SNPs", "REL", "PROD")`
- Do the regression and plot:
 - `lm1 = lm(Hereg$PROD~Hereg$REL)`
 - `png(file="HEreg_all.png")`
 - `plot(Hereg$PROD~Hereg$REL,pch=".")`
 - `abline(lm1,col="orange",lwd=1.5)`
 - `dev.off()`

$$\text{corr}(Y_i, Y_j) = h^2 \boldsymbol{\pi}_{ij} + \text{residual}$$

Practical – Haseman-Elston regression

- Now let's remove the relatives & retry
 - `HEreg$unrel = HEreg$REL < 0.05` #make a T/F vector on the relationship value
 - `lm2 = lm(HEreg$PROD[HEreg$unrel] ~ HEreg$REL[HEreg$unrel])`
 - `plot(HEreg$PROD[HEreg$unrel] ~ HEreg$REL[HEreg$unrel], pch=".")`
 - `abline(lm2, col="orange", lwd=1.5)`
 - `dev.off()`