

Sensitivity analyses in Mendelian randomization studies

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(Some slides adapted from Prof David Evans and Dr Daniel Hwang)



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Acknowledgment of **Country**

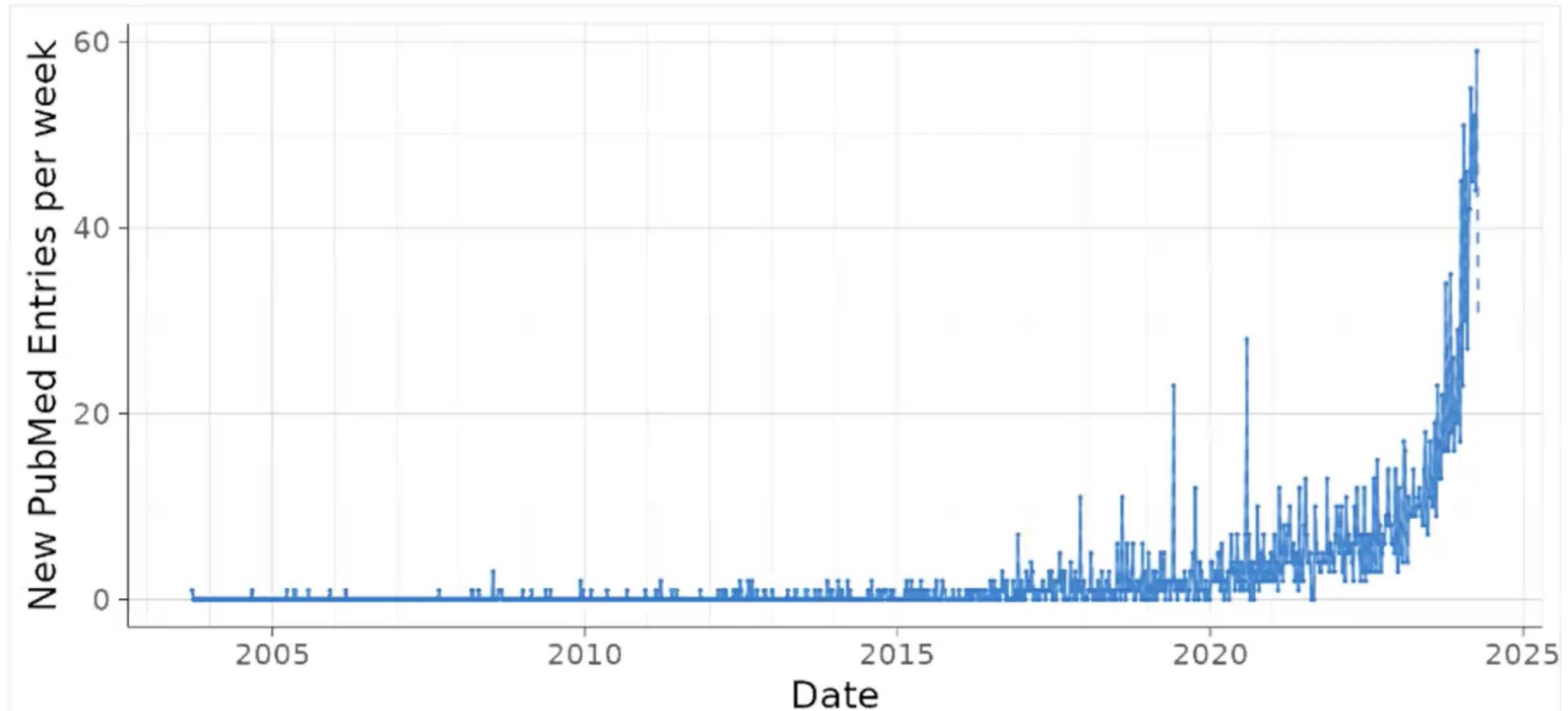
The University of Queensland (UQ) acknowledges the Traditional Owners and their custodianship of the lands on which we meet.

We pay our respects to their Ancestors and their descendants, who continue cultural and spiritual connections to Country.

We recognise their valuable contributions to Australian and global society.



PubMed search for Mendelian randomi[z/s]ation (title only)



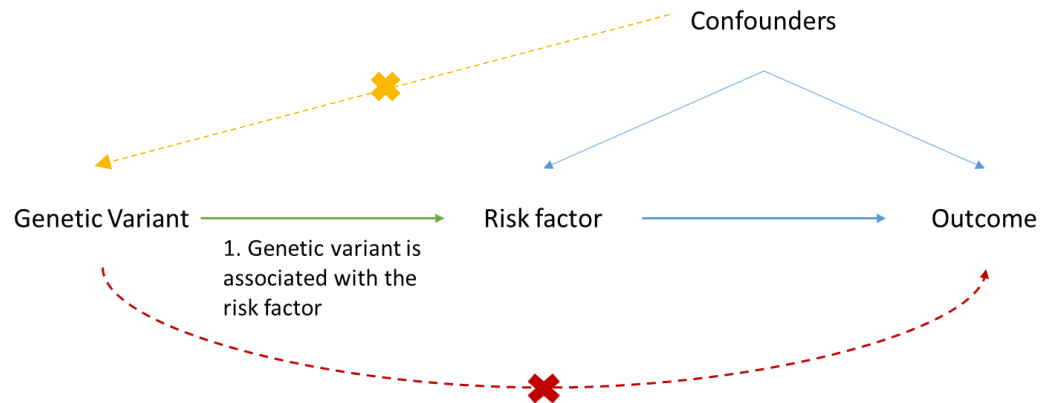
Up to 2024-04-13

Recap

- Mendelian randomization is a technique that uses genetically informative observational data to inform causality
- Three core assumptions:
 - (1) **Relevance assumption:** SNP is associated with the exposure
 - (2) **Independence assumption:** SNP is NOT associated with confounding variables
 - (3) **Exclusion restriction:** SNP ONLY associated outcome through the exposure
- Pleiotropy: Genetic variant influences more than one trait
- One-sample MR is where the SNP, exposure and outcome are all available in the same study
- Two-sample MR is where the SNP-exposure association is measured in one study and the SNP outcome association is measured in a second study

Recap

Assumption 1: Relevance assumption



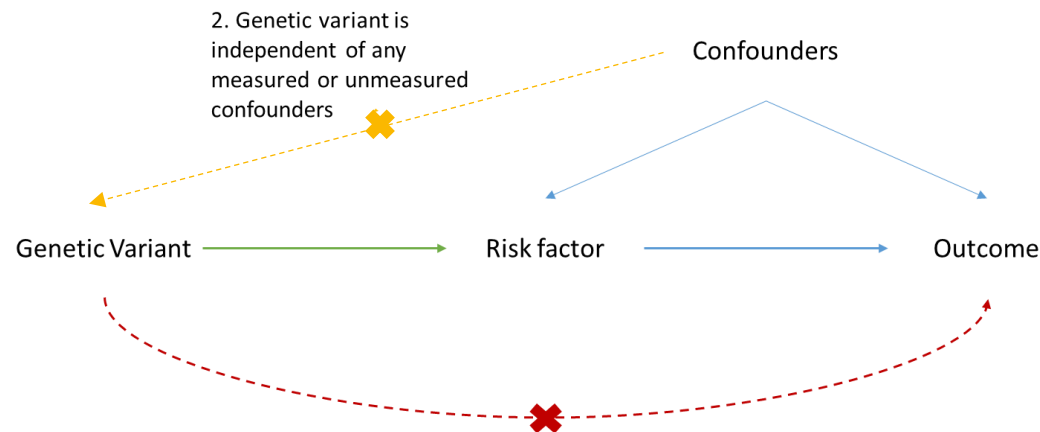
Typically, SNPs which pass genome-wide significance ($P < 5 \times 10^{-8}$) and have been replicated in independent samples are used as IV's

- Weak instruments:
 - Loss of power
 - Bias due to violations of the other assumptions will be amplified
 - Bias towards outcome-risk factor association in one-sample MR or towards the null in two-sample MR – precision is also underestimated.
- Weak instruments can be detected using an F-statistic in one-sample MR (F-statistic > 10)

$$F_{\text{stat}} = \frac{R^2 * (N-1)}{(1-R^2)}$$

Recap

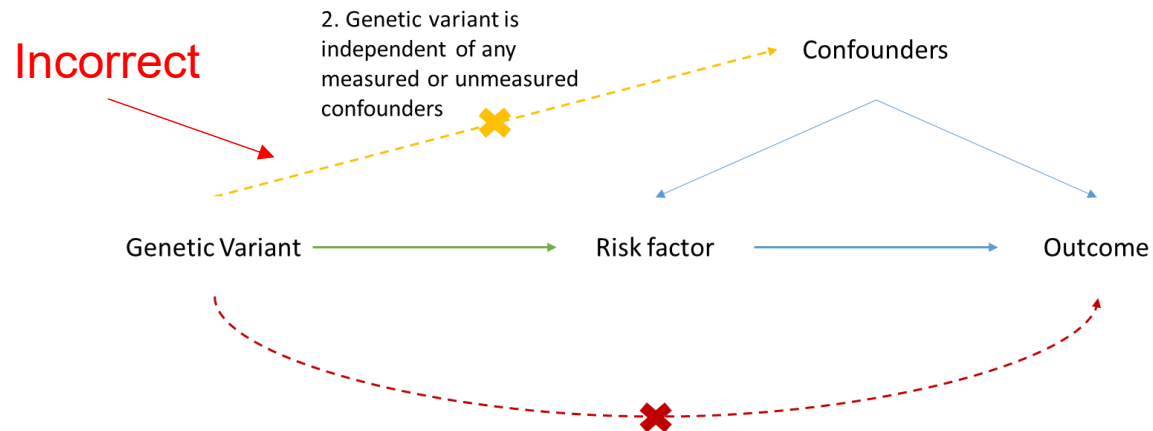
Assumption 2: Independence assumption



- Technically impossible to prove this assumption holds as we can't test for association with *unobserved* confounders (need to rely on good knowledge of the science)
- May be possible to disprove by checking that the genetic variant is independent of *measured* confounders of the exposure-outcome relationship
- Factors that could influence the genetic variants and outcome include population stratification or structure, intergenerational (dynastic) effects and assortative mating.

Recap

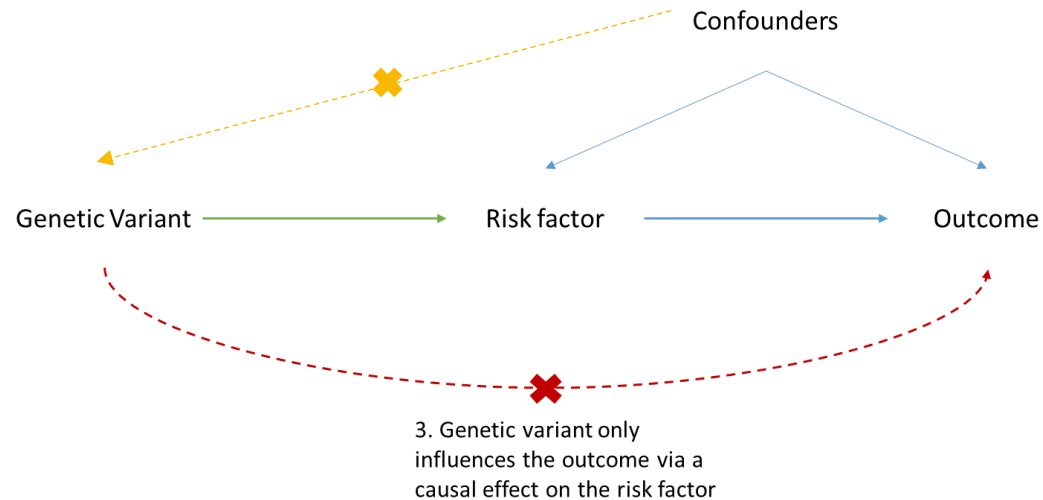
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Recap

Assumption 3: Exclusion restriction



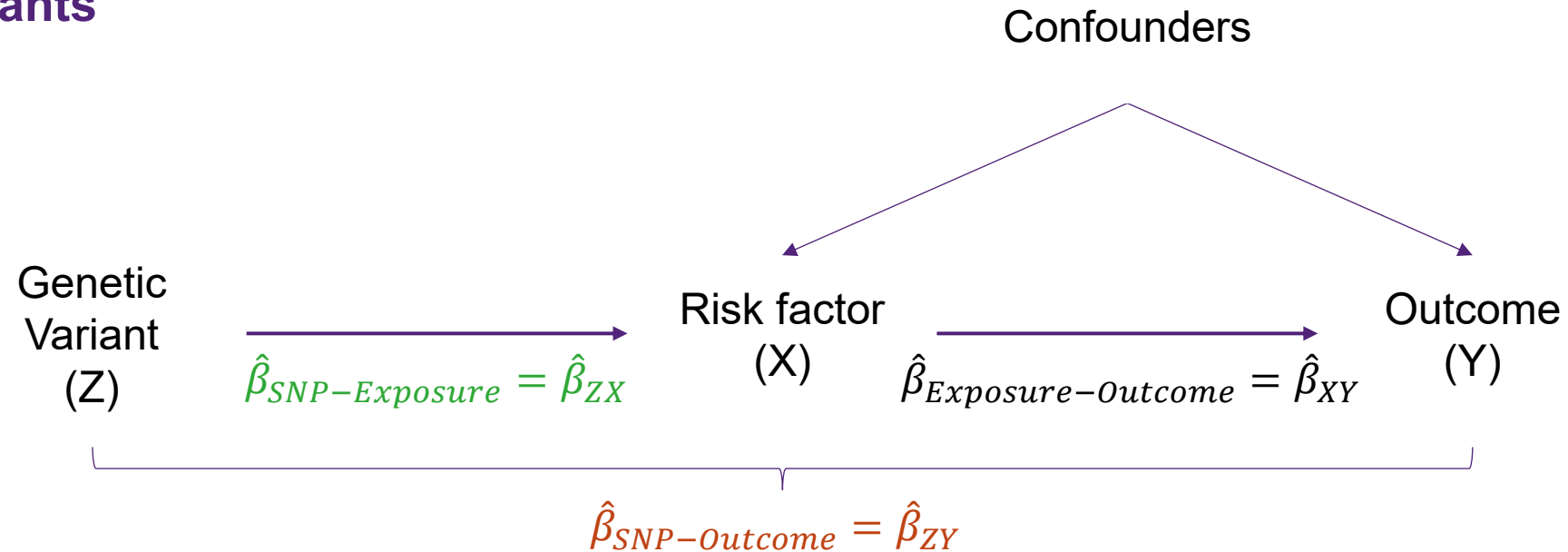
- Again, is difficult to prove this assumption holds
- Horizontal pleiotropy = SNP is associated with multiple traits independently of the exposure of interest
- Extensions to the basic MR design can be used to detect horizontal pleiotropy and estimate causal effect in its presence

Outline

- Inverse variance weighted MR
- Heterogeneity tests
- Multivariable MR
- MR Egger
- MR Weighted Median
- Steiger Filtering

Two-sample MR

Single variants



Causal effect ($\hat{\beta}_{XY}$) by Wald estimator: $\frac{\hat{\beta}_{SNP-Outcome}}{\hat{\beta}_{SNP-Exposure}}$

Standard error ($\hat{\sigma}_{XY}$) by Delta method: $\frac{\sigma_{SNP-Outcome}}{\hat{\beta}_{SNP-Exposure}}$

$$\hat{\beta}_{SNP-Outcome} = \hat{\beta}_{SNP-Exposure} \times \hat{\beta}_{Exposure-Outcome}$$

Can be estimated in different samples (e.g. two-sample MR)

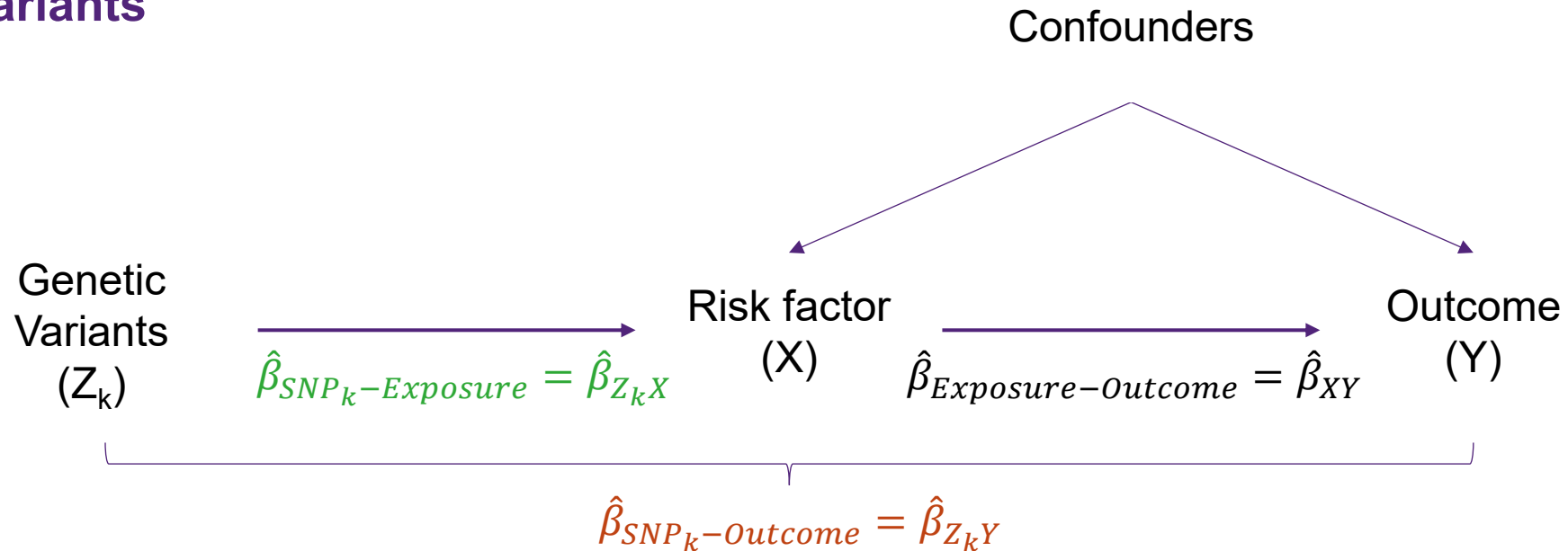
Delta method to estimate SE of Wald ratio

$$\begin{aligned}
 \text{Var}(\hat{\beta}_{xy}) &= \text{Var}\left(\frac{\hat{\beta}_{\text{SNP-Outcome}}}{\hat{\beta}_{\text{SNP-Exposure}}}\right) \\
 &\approx \frac{\text{Var}(\hat{\beta}_{\text{SNP-Outcome}})}{\hat{\beta}_{\text{SNP-Exposure}}^2} + \left(\frac{\hat{\beta}_{\text{SNP-Outcome}}^2}{\hat{\beta}_{\text{SNP-Exposure}}^4}\right) \text{Var}(\hat{\beta}_{\text{SNP-Exposure}}) - 2\left(\frac{\hat{\beta}_{\text{SNP-Outcome}}}{\hat{\beta}_{\text{SNP-Exposure}}^3}\right) \text{Cov}(\hat{\beta}_{\text{SNP-Exposure}}, \hat{\beta}_{\text{SNP-Outcome}}) \\
 &\approx \frac{\text{Var}(\hat{\beta}_{\text{SNP-Outcome}})}{\hat{\beta}_{\text{SNP-Exposure}}^2}
 \end{aligned}$$

$$\begin{aligned}
 \text{SE}(\hat{\beta}_{XY}) = \hat{\sigma}_{XY} &\approx \sqrt{\frac{\text{Var}(\hat{\beta}_{\text{SNP-Outcome}})}{\hat{\beta}_{\text{SNP-Exposure}}^2}} \\
 &\approx \frac{\sigma_{\text{SNP-Outcome}}}{\hat{\beta}_{\text{SNP-Exposure}}}
 \end{aligned}$$

Two-sample MR

Multiple variants



Causal effect by Wald estimator:

$$\hat{\beta}_{XY_k} = \frac{\hat{\beta}_{SNP_k-Outcome}}{\hat{\beta}_{SNP_k-Exposure}}$$

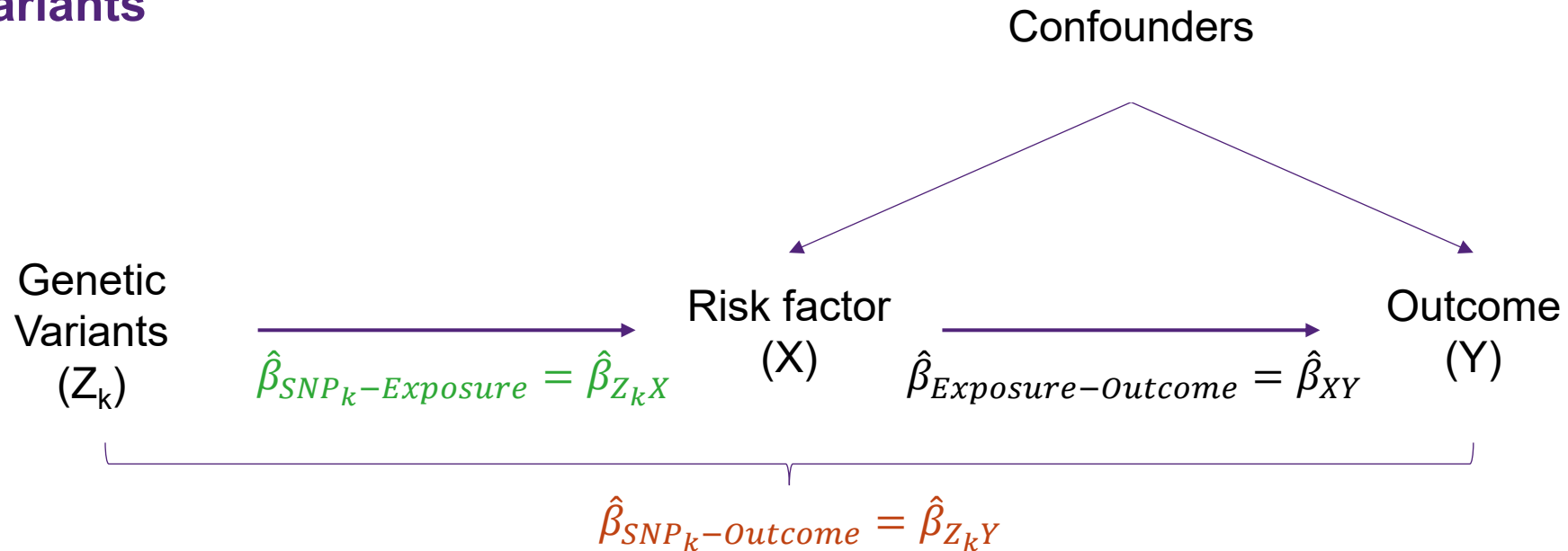
Inverse variance weighted (IVW) average causal effect:

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is the inverse variance of the causal effect estimated from the k^{th} genetic variant

Two-sample MR

Multiple variants



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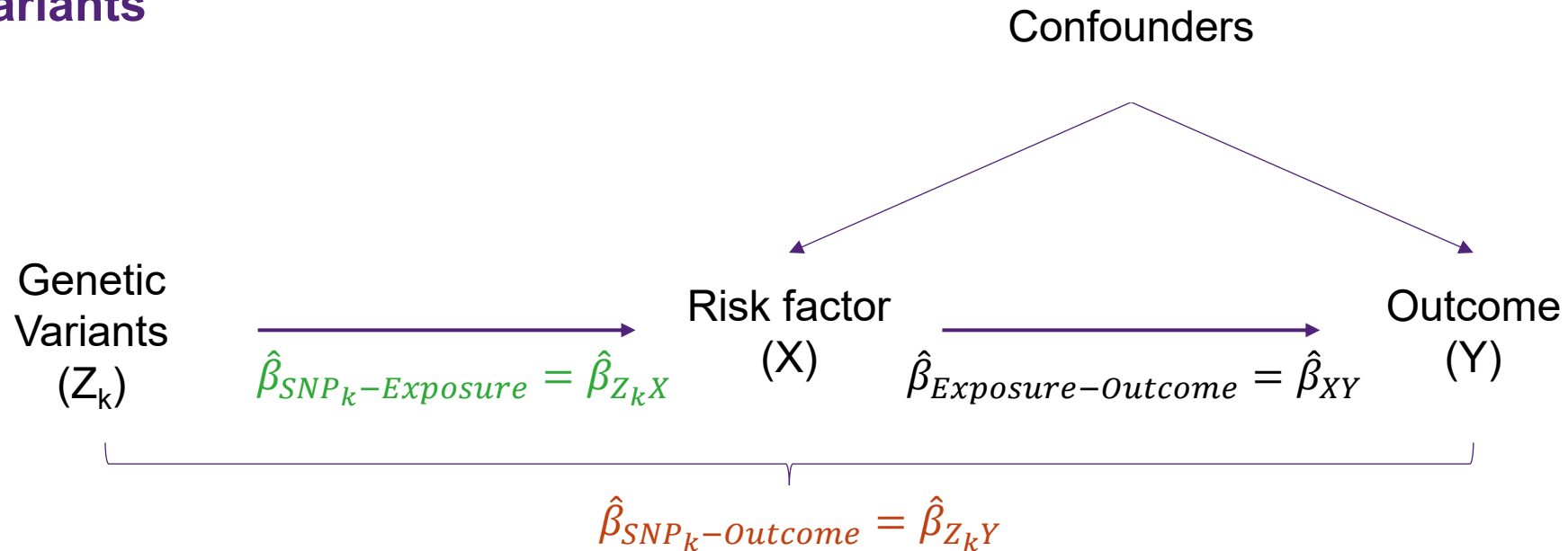
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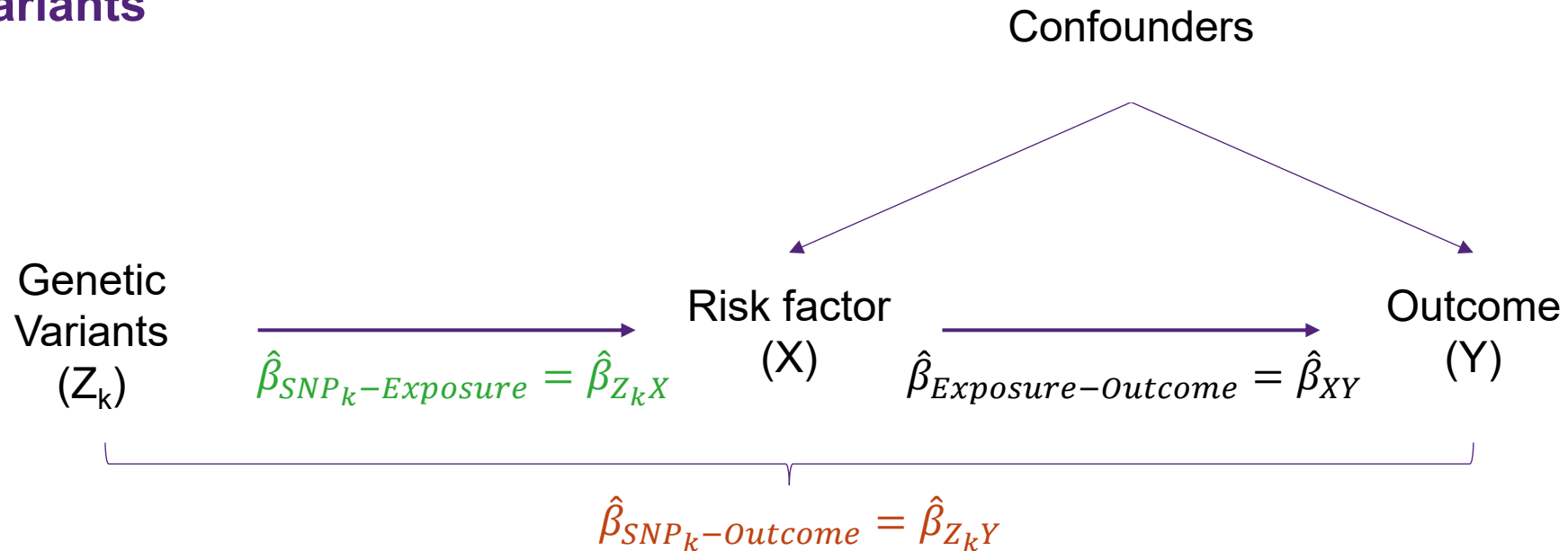
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Two-sample MR

Multiple variants



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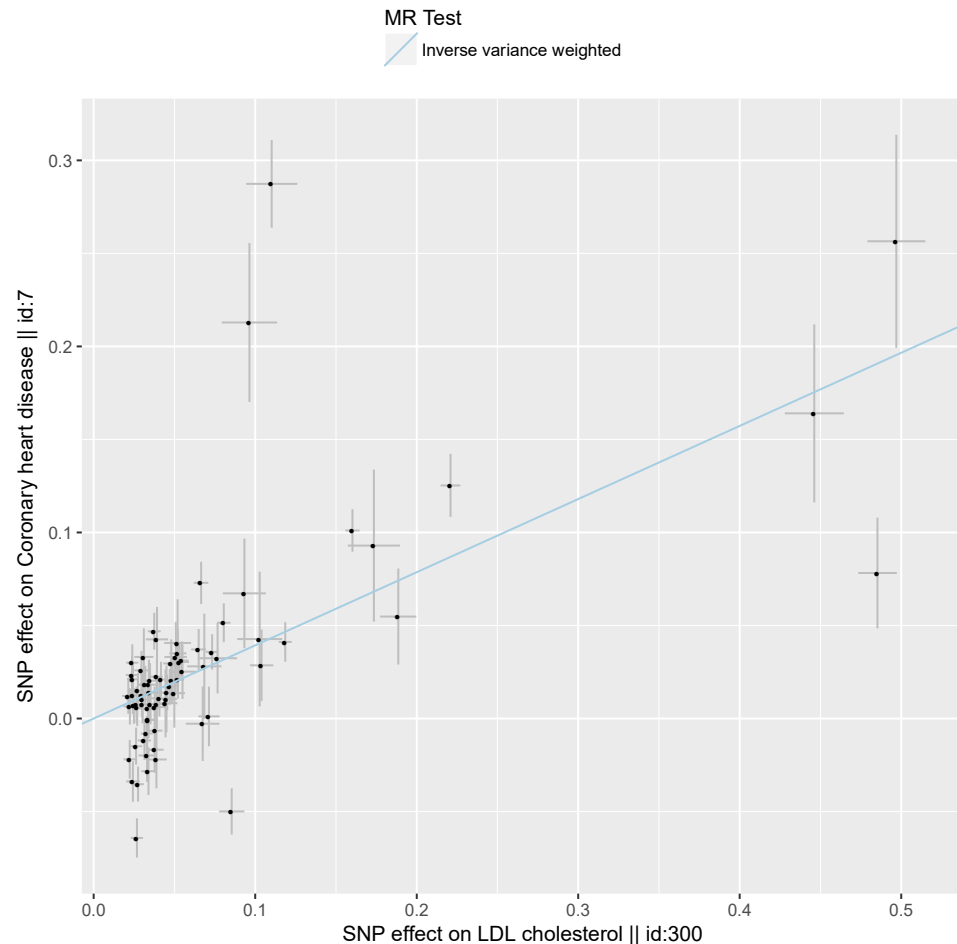
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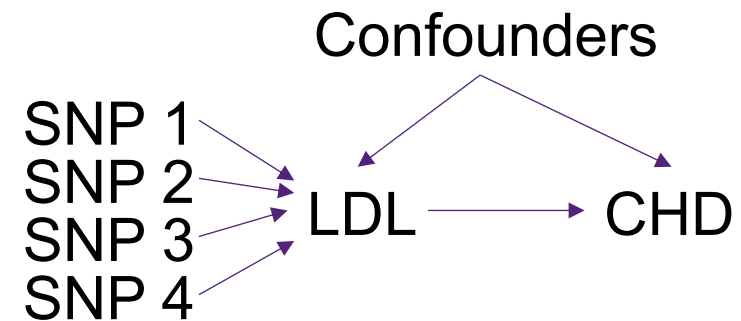
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Fixed effects IVW-MR and weighted linear regression



- IVW is equivalent to a weighted regression of SNP-outcome effects on SNP-exposure effects passing through the origin
- The weights are $\frac{1}{\sigma_{Z_k Y}^2}$
- The slope is the estimate of the causal effect



Assumptions for two-sample MR

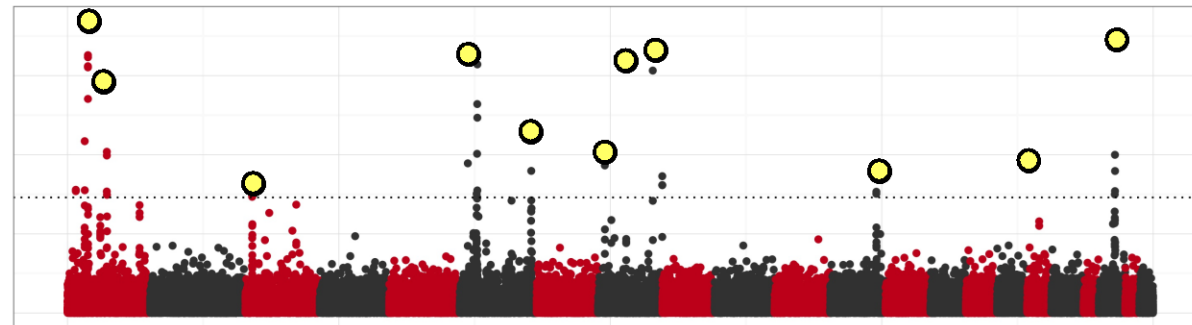
- Using summarized data for two-sample MR analyses is convenient when sharing individual level data is impractical
- If:
 - The K genetic variants are perfectly uncorrelated (not in LD) and do not interact
 - The two samples are homogenous (same underlying populations)
 - Constant causal effect at each level of the exposure

Then two-sample MR can consistently estimate the true causal effect

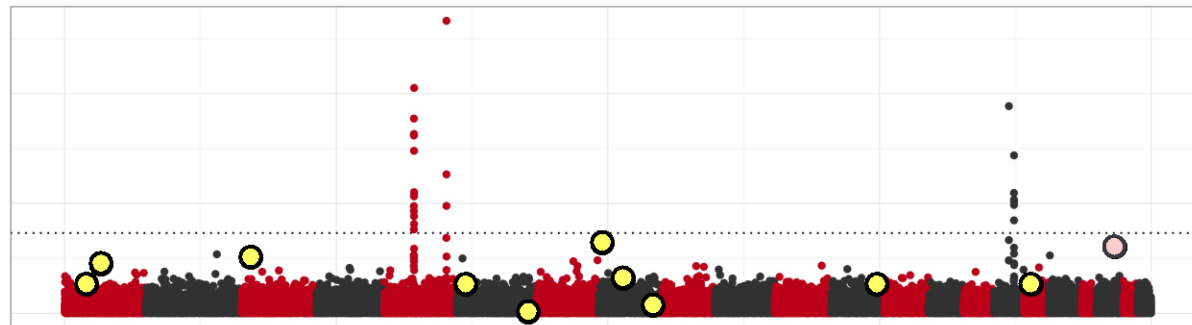
- Two-sample MR is still vulnerable to weak instrument bias
 - Bias towards the null effect, not the observational estimate
 - If approximate F-statistic ($\hat{\beta}_{Z_kX}^2 / \sigma_{Z_kX}^2$) is greater than 10, then the expected dilution of $\hat{\beta}_{XY_k}$ towards zero is less than 10%

Performing MR with summary statistics

Obtain instruments from
exposure GWAS

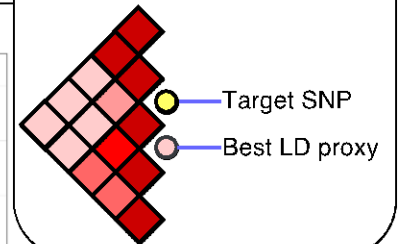


Extract SNP effects from
outcome GWAS

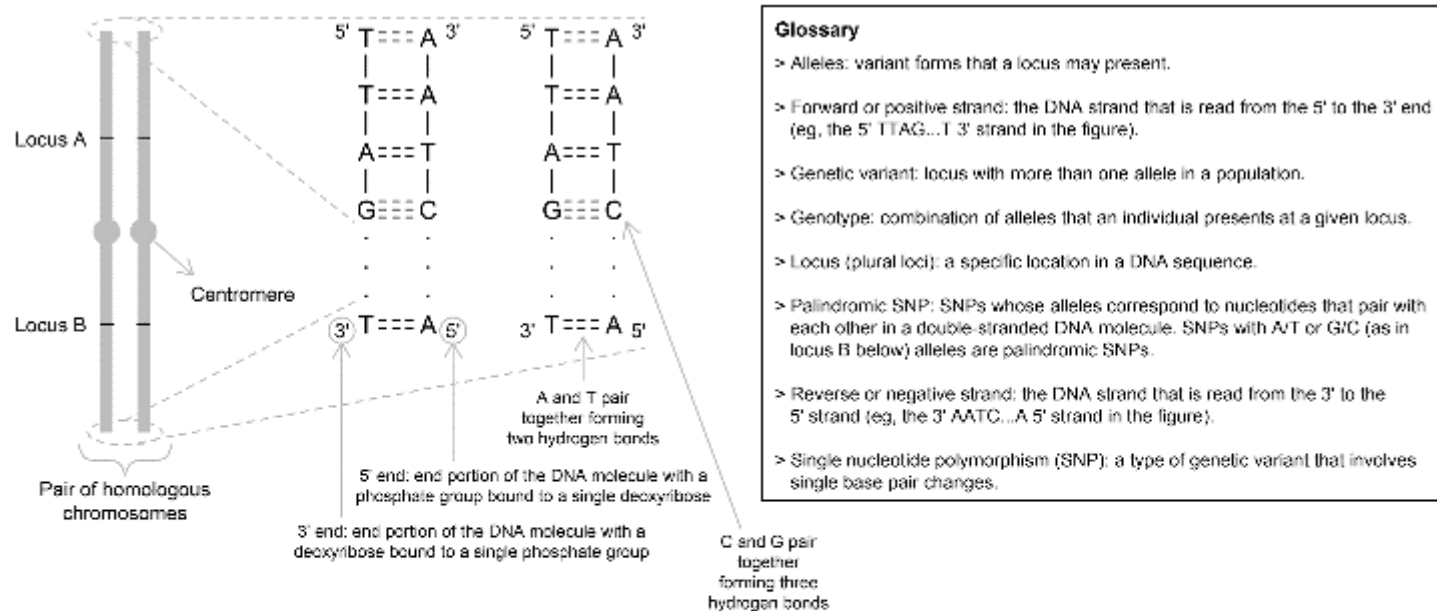


LD Proxies

If an exposure instrument is not available in the outcome GWAS then look for LD proxies in 1000 genomes



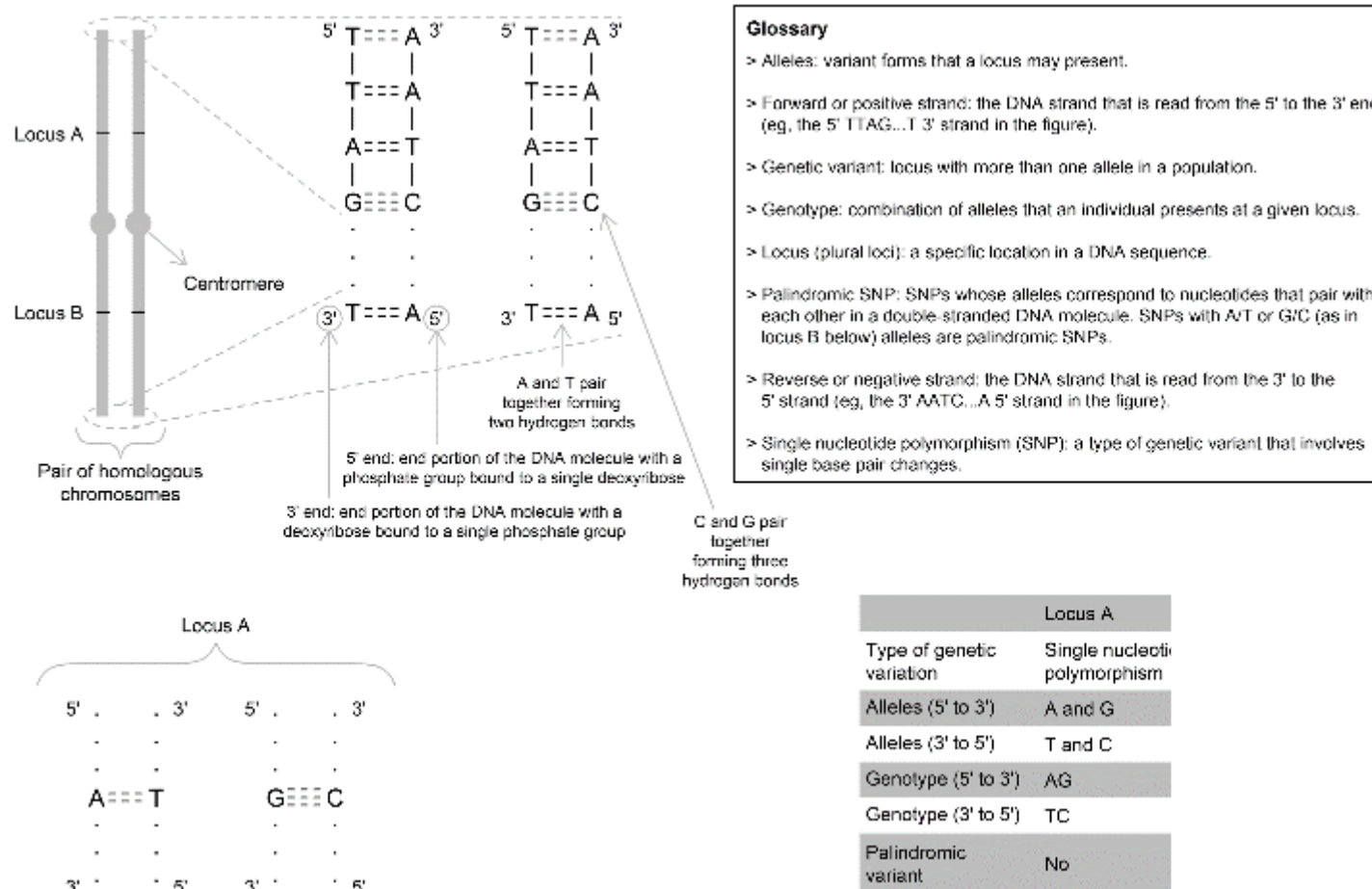
The issue of strand



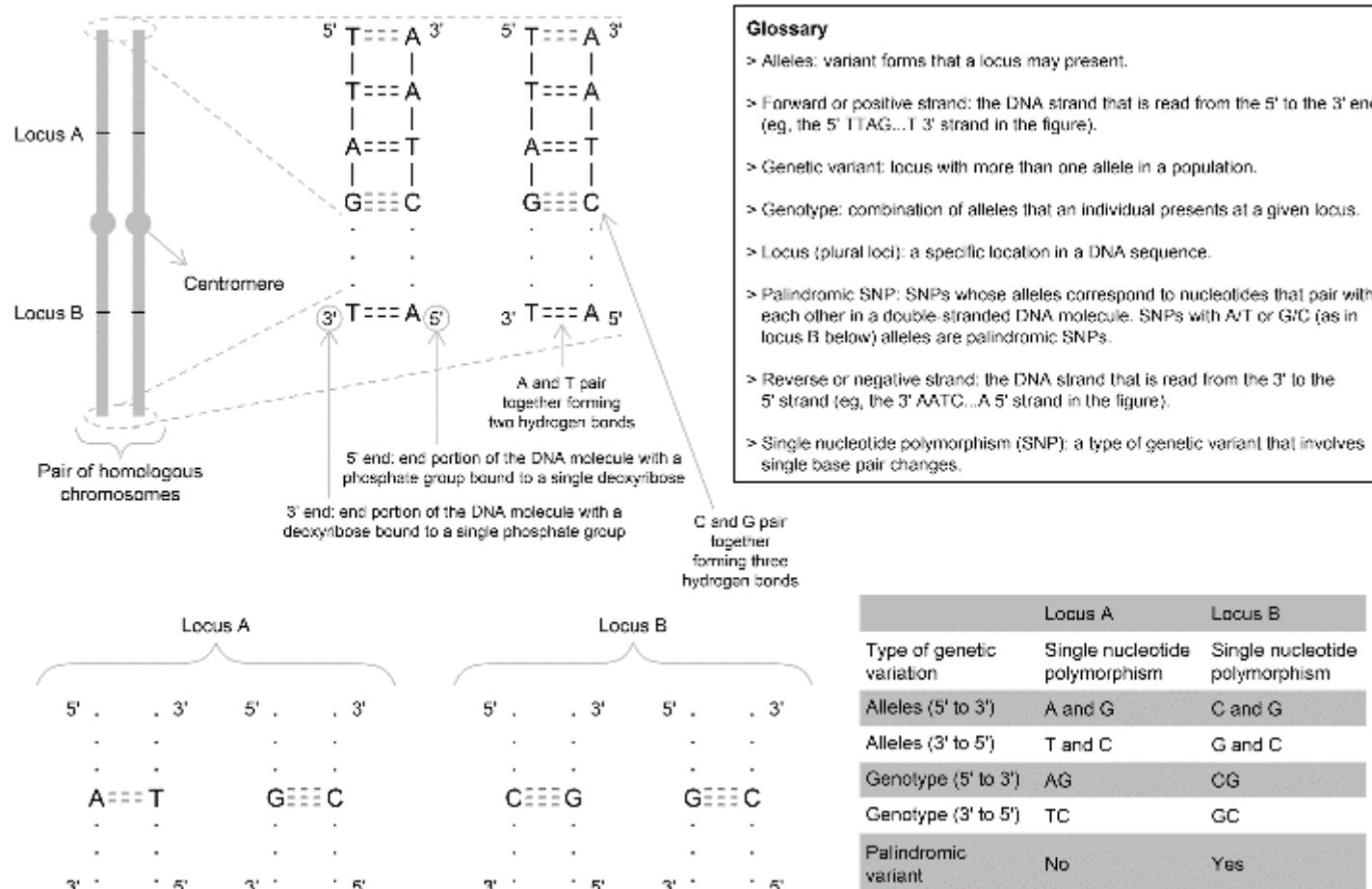
Glossary

- > Alleles: variant forms that a locus may present.
- > Forward or positive strand: the DNA strand that is read from the 5' to the 3' end (eg, the 5' TTAG...T 3' strand in the figure).
- > Genetic variant: locus with more than one allele in a population.
- > Genotype: combination of alleles that an individual presents at a given locus.
- > Locus (plural loci): a specific location in a DNA sequence.
- > Palindromic SNP: SNPs whose alleles correspond to nucleotides that pair with each other in a double-stranded DNA molecule. SNPs with A/T or G/C (as in locus B below) alleles are palindromic SNPs.
- > Reverse or negative strand: the DNA strand that is read from the 3' to the 5' strand (eg, the 3' AATC...A 5' strand in the figure).
- > Single nucleotide polymorphism (SNP): a type of genetic variant that involves single base pair changes.

The issue of strand

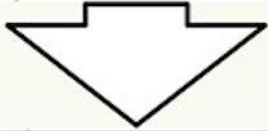


The issue of strand



Harmonise exposure and outcome effects

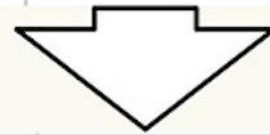
Exposure GWAS					Outcome GWAS			
SNP	Effect	Effect allele	Other allele	Effect allele frequency	Effect	Effect allele	Other allele	Effect allele frequency
rs12345	0.132	A	G	0.28	0.022	A	G	0.26
rs23456	-0.485	G	T	0.41	0.056	T	G	0.61
rs34567	0.203	G	C	0.11	-0.046	G	C	0.88



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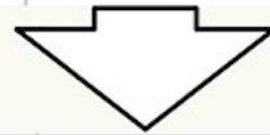
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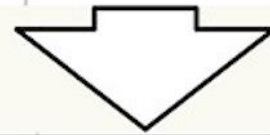
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Harmonise exposure and outcome effects

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rs34567	0.203	G	C	0.11	0.046	G	C	0.12

Strand issue exercise

SNP	Study 1 alleles	Study 1 allele freq	Study 2 alleles	Study 2 allele freq	Verdict?
rs1	A/G	0.2	A/G	0.2	
rs2	G/T	0.3	T/G	0.72	
rs3	G/C	0.65	G/C	0.62	
rs4	A/T	0.49	A/T	0.5	
rs5	A/T	0.12	A/T	0.89	
rs6	A/G	0.4	A/T	0.4	

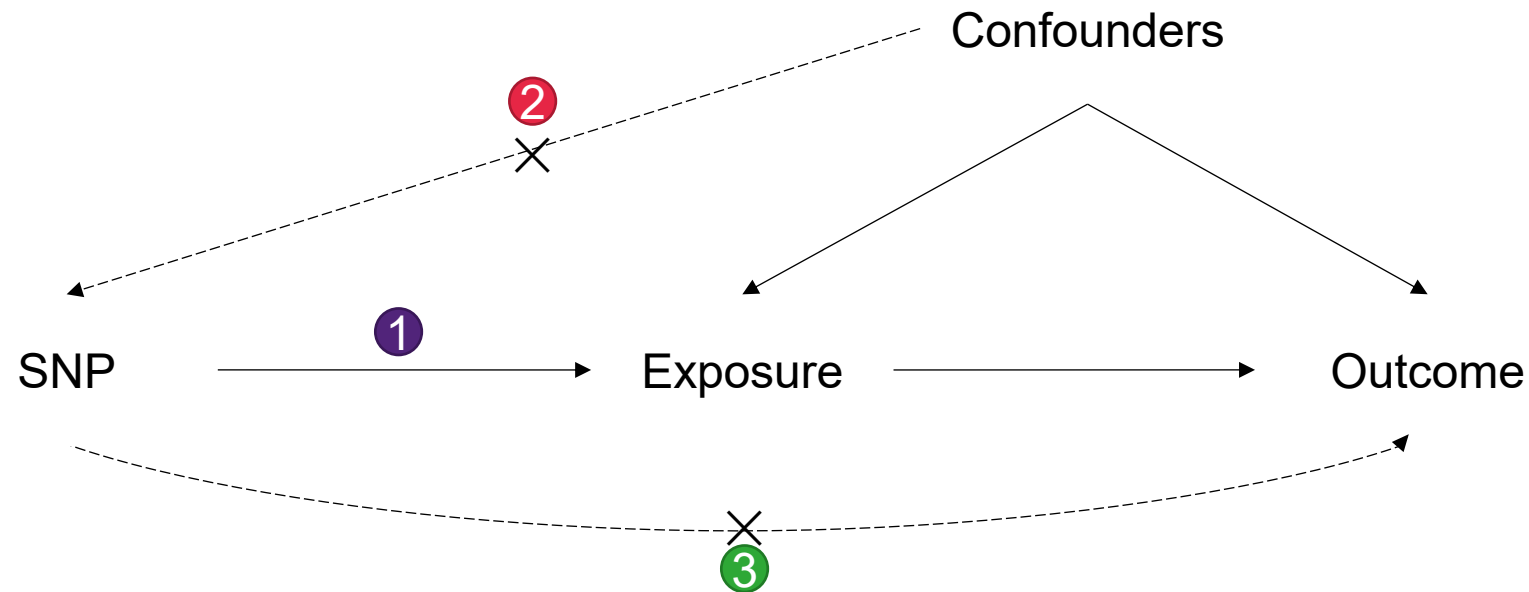
MR methods for handling horizontal pleiotropy

Many methods now exist!

Extensions to MR

- MR uses genetic variants to test for causal relationships between phenotypic exposures and disease-related outcomes
- Due to the proliferation of GWAS, it is increasingly common for MR analyses to use large numbers of genetic variants
- Increased power but greater potential for pleiotropy
- Pleiotropic variants affect biological pathways other than the exposure under investigation and therefore can lead to biased causal estimates and false positives under the null

Two-sample MR



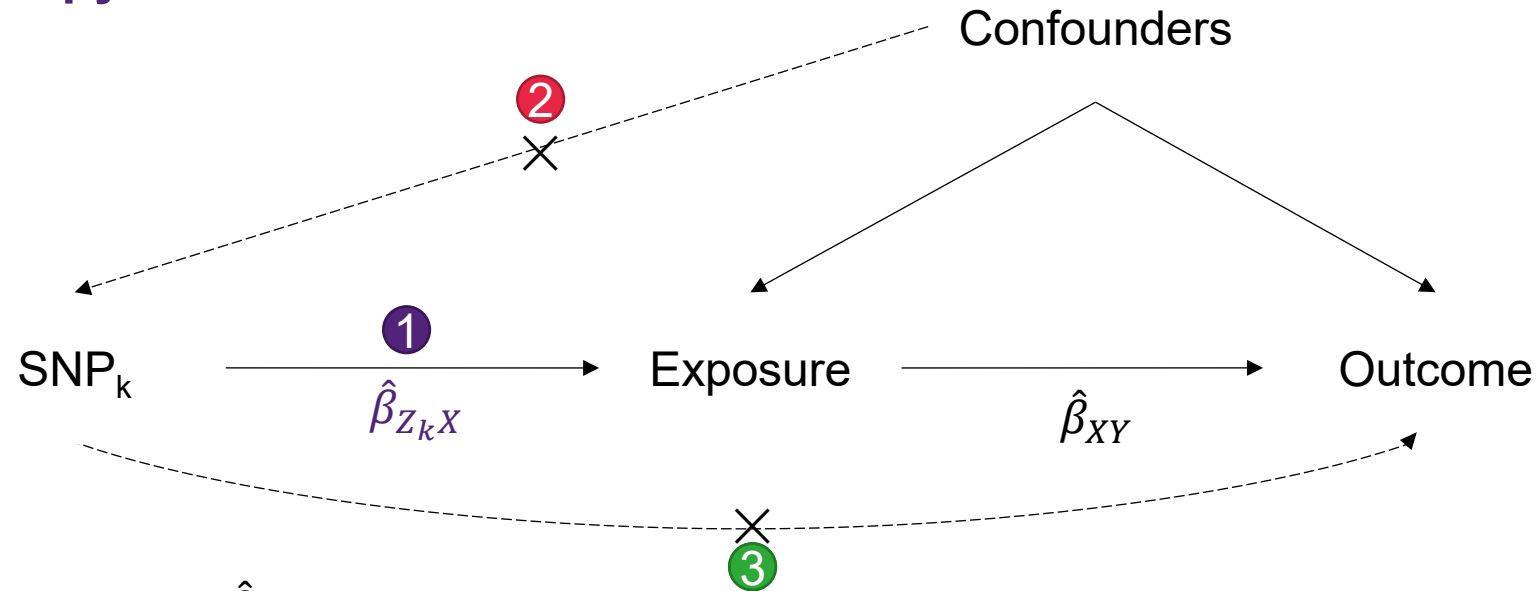
(1) Relevance assumption: SNP is associated with the exposure

👉 (2) Independence assumption: SNP is NOT associated with confounding variables

👉 (3) Exclusion restriction: SNP ONLY associated outcome through the exposure

Two-sample MR

No direct pleiotropy



$$\hat{\beta}_{SNP-Outcome} = \hat{\beta}_{SNP-Exposure} \times \hat{\beta}_{Exposure-Outcome}$$

Causal effect by Wald estimator:

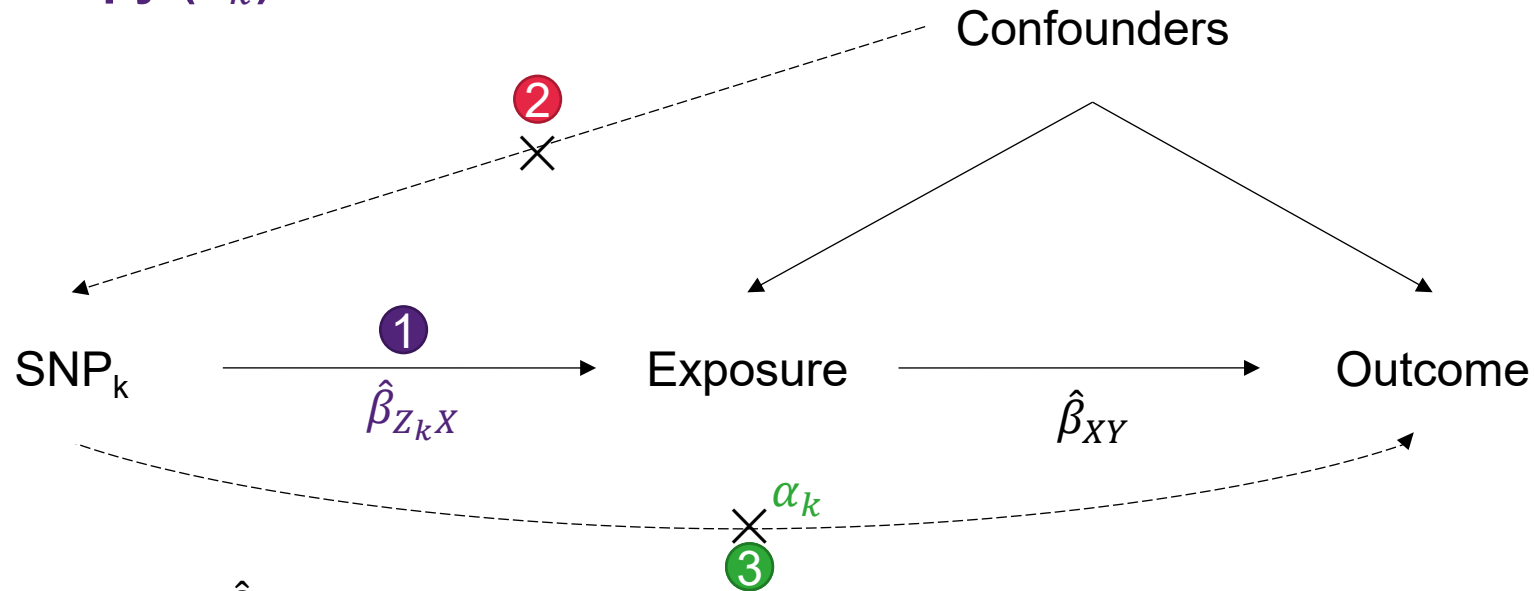
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Inverse variance weighted (IVW) average causal effect:

$$\hat{\beta}_{IVW} = \frac{\sum_{k=1}^K \hat{\beta}_{XY_k} w_k}{\sum_{k=1}^K w_k}$$

Two-sample MR

With direct pleiotropy (α_k)



$$\hat{\beta}_{SNP-Outcome} = \hat{\beta}_{SNP-Exposure} \times \hat{\beta}_{Exposure-Outcome}$$

Causal effect by Wald estimator:

$$\frac{\hat{\beta}_{SNP_k-Outcome}}{\hat{\beta}_{SNP_k-Exposure}} = \hat{\beta}_{XY_k} + \frac{\alpha_k}{\hat{\beta}_{SNP_k-Exposure}}$$

Inverse variance weighted
(IVW) average causal effect:

$$\frac{\sum_{k=1}^K \hat{\beta}_{XY_k} w_k}{\sum_{k=1}^K w_k} = \hat{\beta}_{IVW} + \text{Bias}(\alpha, \hat{\beta}_{SNP_k-Exposure})$$

Heterogeneity

We expect that each SNP represents an independent study, and each should give an unbiased (if imprecise) estimate of the causal effect of X on Y.

Heterogeneity, where effect estimates are more different than expected, arises because at least some of the instruments are invalid.

Cochran's Q statistic (heterogeneity test):

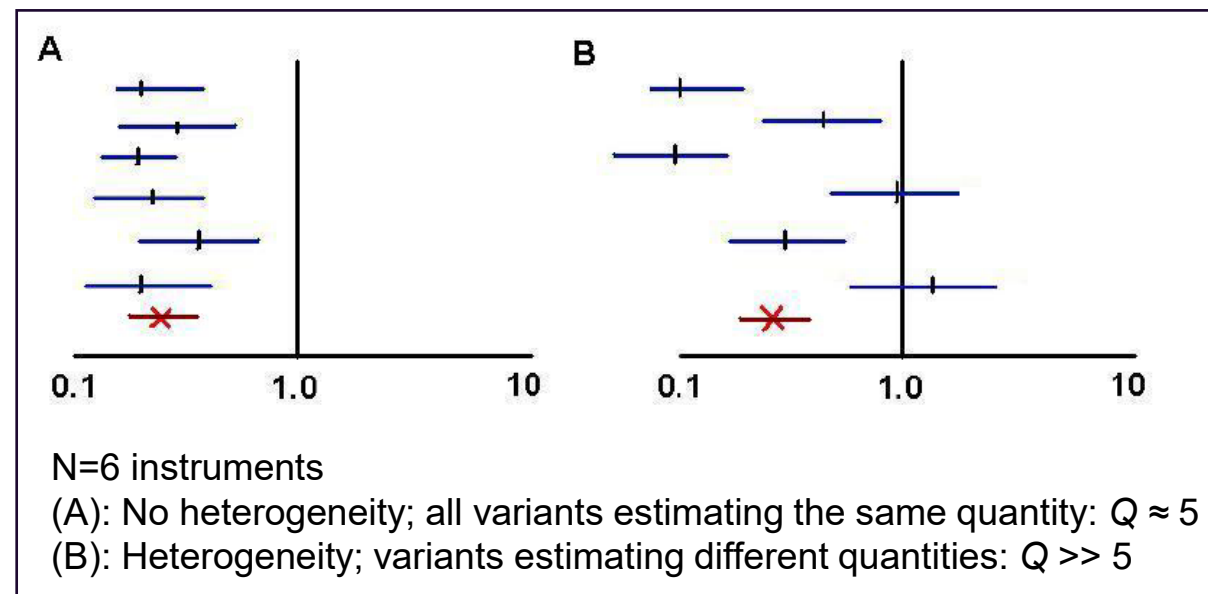
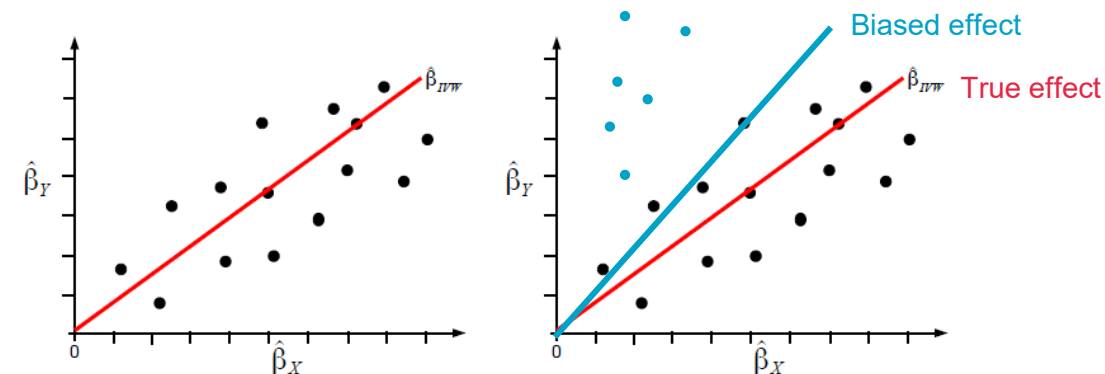
$$Q = \sum_{k=1}^K \frac{1}{v_k} (\hat{\beta}_{XY_k} - \hat{\beta}_{IVW})^2$$

Where v_k is the variance of the causal estimate at SNP k

If MR model is correct, Q follows a χ^2 distribution with expected value $K-1$.

If Q is larger than $K-1$, then it's plausible that there are one or more genetic variants that have pleiotropic effects.

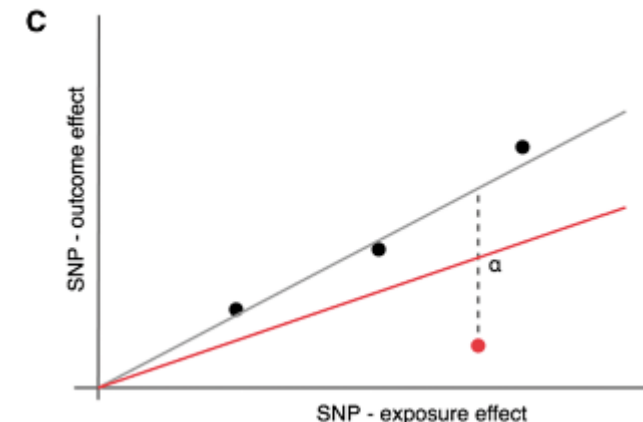
- SNPs are valid instruments
- SNPs associated with outcome via an independent pathway.



Accounting for heterogeneity

Option 1: Remove outliers

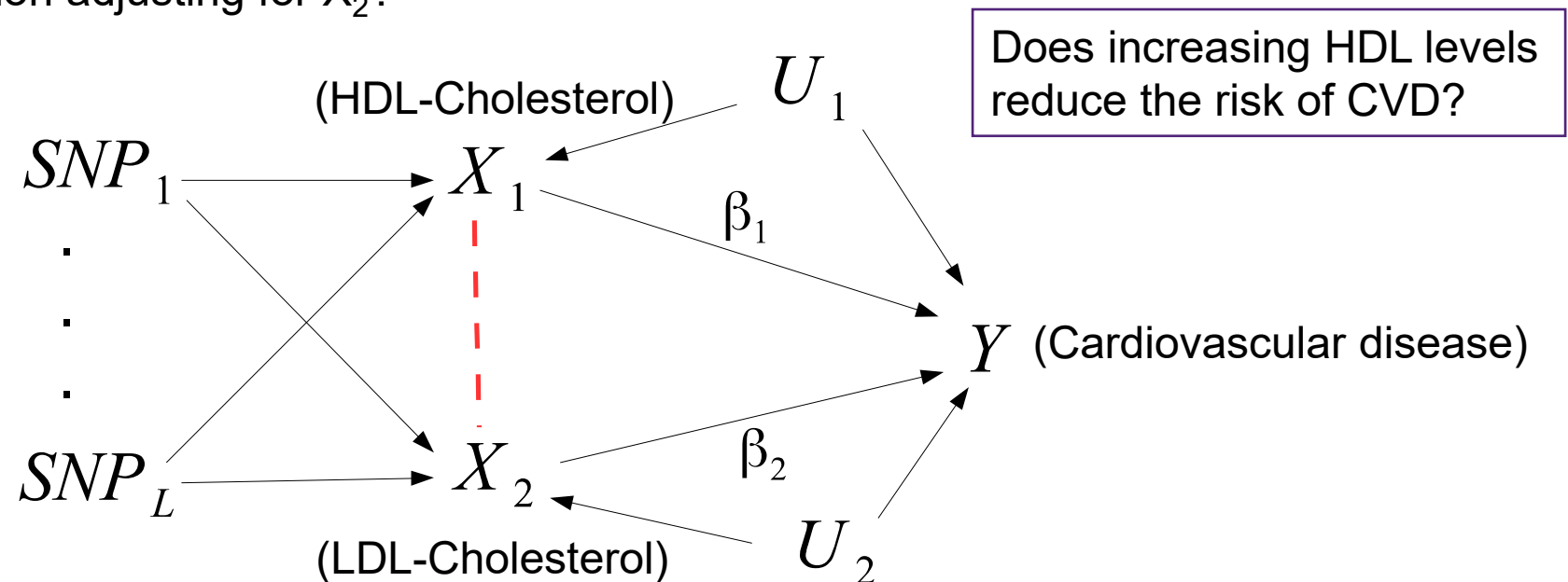
- Some SNPs might contribute to the majority of the heterogeneity
- If we assume these are the invalid instruments, then the IVW estimate excluding them should be less biased
- However – beware of:
 - Cherry picking – removing outliers will artificially provide a more precise estimate
 - What if the outlier is the only valid instrument, and all the others are invalid?
 - E.g. cis-variants for gene expression, DNA methylation, protein levels.
 - CRP levels are best instrumented by variants within the CRP gene region. Most other variants that come up in CRP GWAS are upstream effects related to inflammation



Accounting for heterogeneity

Option 2: Multivariable MR

- We are testing for whether X_1 has an influence on Y
- We know that some instruments for X_1 also have influences on X_2
- This opens up the possibility of horizontal pleiotropy biasing our estimate
- What is the X_1 - Y association adjusting for X_2 ?



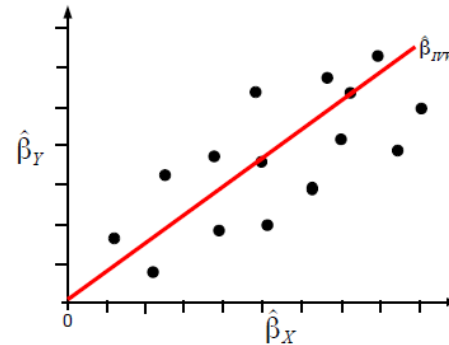
Accounting for heterogeneity

Option 3: Fit a model that is robust to some model of horizontal pleiotropy

- IVW fixed effects estimate assumes all SNPs are valid instruments, and averages across them all
- Additive random effects estimate:
 - Estimate the between IV estimate of heterogeneity (denoted by τ^2), then calculate and update IVW estimate by replacing v_k with $v_k + \tau^2$
 - Point estimate and variance different from $\hat{\beta}_{IVW}$
- Multiplicative random effects model
 - Replace v_k with ϕv_k , where $\phi = \frac{Q}{K-1}$
 - Point estimate equals $\hat{\beta}_{IVW}$, but variance is inflated
- Additive random effects model popular in meta-analysis, but can perform poorly in the presence of pleiotropy

Heterogeneity and pleiotropy

- IVW assumes all variants are valid instrumental variables
 - Clear trend in estimates increasing with $\hat{\beta}_{Z_kX}$ from origin
 - Cochran's $Q \approx K - 1$ (no heterogeneity)



- If there is an indication that these don't hold in the data, invalid “pleiotropic” variants could be the cause

Investigating heterogeneity and pleiotropy

- The IVW method assumes the underlying SNP-outcome model is

$$\hat{\beta}_{Yk} = \beta_{IVW}\beta_{Xk} + \varepsilon_{Yk} \quad (\varepsilon_{Yk} \text{ independent of } \hat{\beta}_{Xk})$$

β_{Xk} replaced with $\hat{\beta}_{Xk}$ when fitting the model

- A more realistic model to account for heterogeneity might be:

$$\hat{\beta}_{Yk} = \alpha_k + \beta_{IVW}\beta_{Xk} + \varepsilon_{Yk}$$

Where α_k is the pleiotropic effect of variant k

- Can the IVW method still estimate the causal effect without bias even when all variants have pleiotropic effects? Yes, if:
 - α_k is independent of β_{Xk} across K SNPs (InSIDE assumption)
 - The mean value of α_k is zero
 - If satisfied, pleiotropy is said to be balanced

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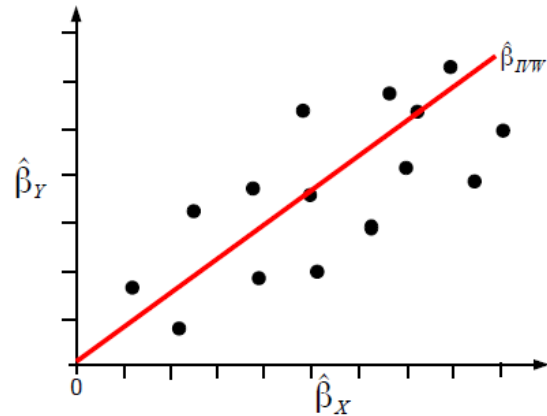
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$$\hat{\beta}_{Yk} = \alpha_k + \beta_{IVW}\beta_{Xk} + \varepsilon_{Yk}$$

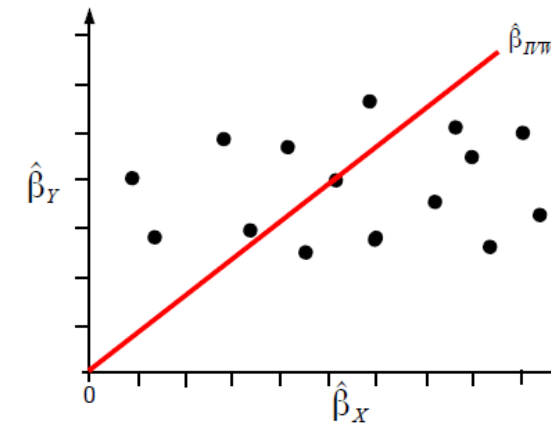
Where α_k is the pleiotropic effect of variant k

- Can the IVW method still estimate the causal effect without bias even when all variants have pleiotropic effects? Yes, if:
 - α_k is independent of β_{Xk} across K SNPs (InSIDE assumption)
 - The mean value of α_k is zero
 - If satisfied, pleiotropy is said to be balanced

Balanced or directional pleiotropy



- Trend towards origin + heterogeneity
- Pleiotropy potentially causing heterogeneity -> IVW appears to be a good fit

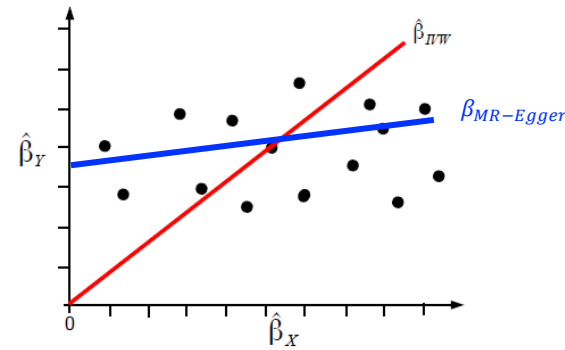


- Trend away from origin + heterogeneity
- Pleiotropy potentially causing heterogeneity and bias
 - IVW does not appear to be good fit
 - Zero-intercept condition unreasonable

MR-Egger regression: Central concept

We could therefore regress the SNP-outcome associations on the SNP-exposure associations, but allow for a non-zero intercept in the regression

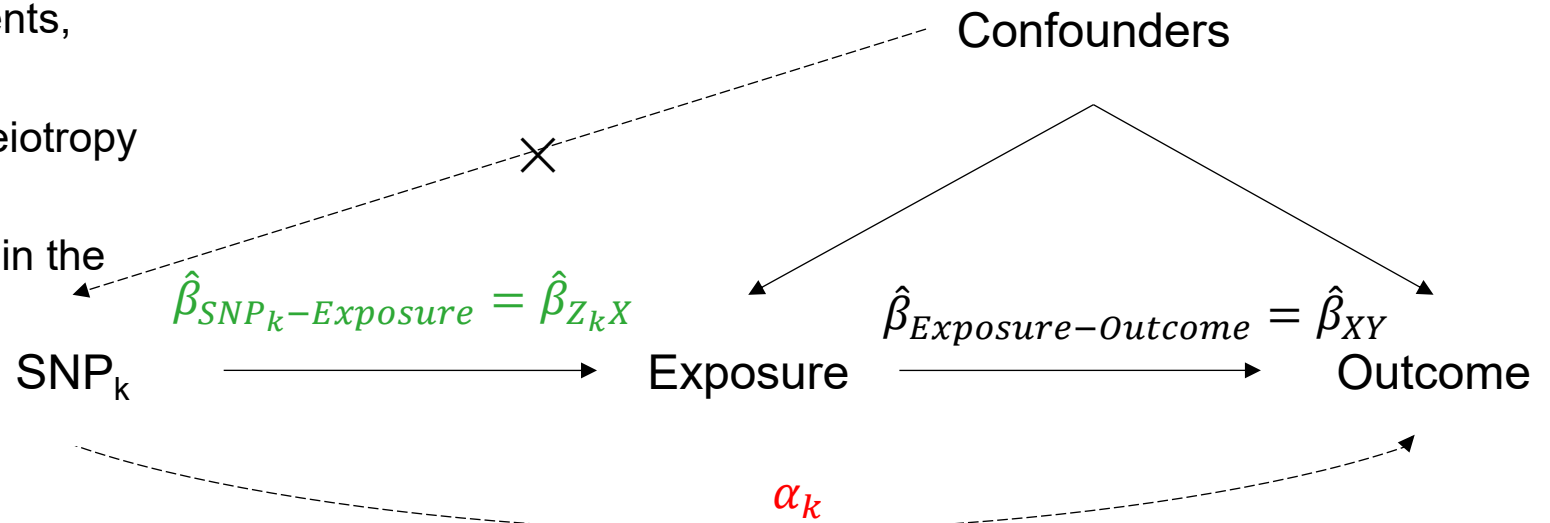
MR-Egger allows for a **non-zero intercept** in the regression.



When multiple SNPs are used as instruments, MR-Egger can:

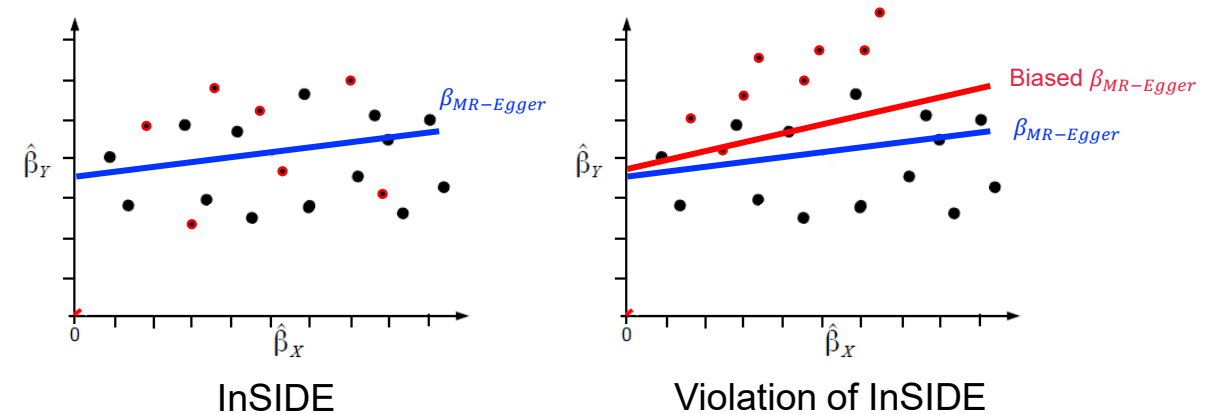
- Identify the presence of “directional” pleiotropy (biasing the causal estimate in IVW)
- Provide a less biased causal estimate (in the presence of pleiotropy)

MR-Egger lacks power.



MR-Egger regression

MR-Egger regression relies on the InSIDE (INstrument Strength Independent of Direct Effect) assumption, which states that the pleiotropic effects of SNPs must be independent of their strength as instruments.

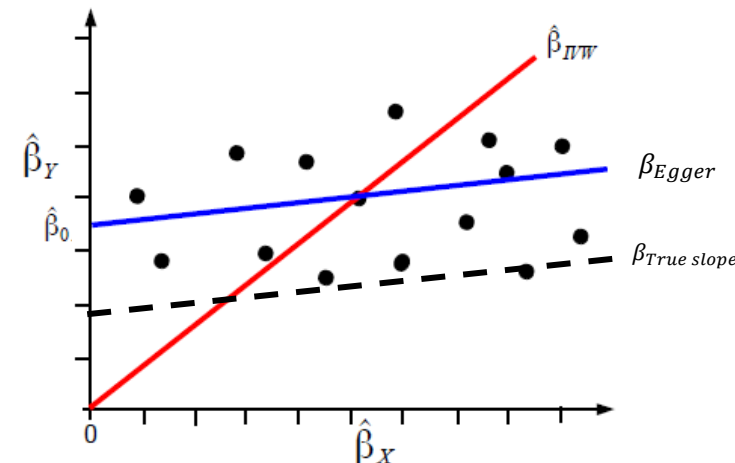


- SNP not associated with outcome via an independent pathway
- SNP associated with outcome via an independent pathway

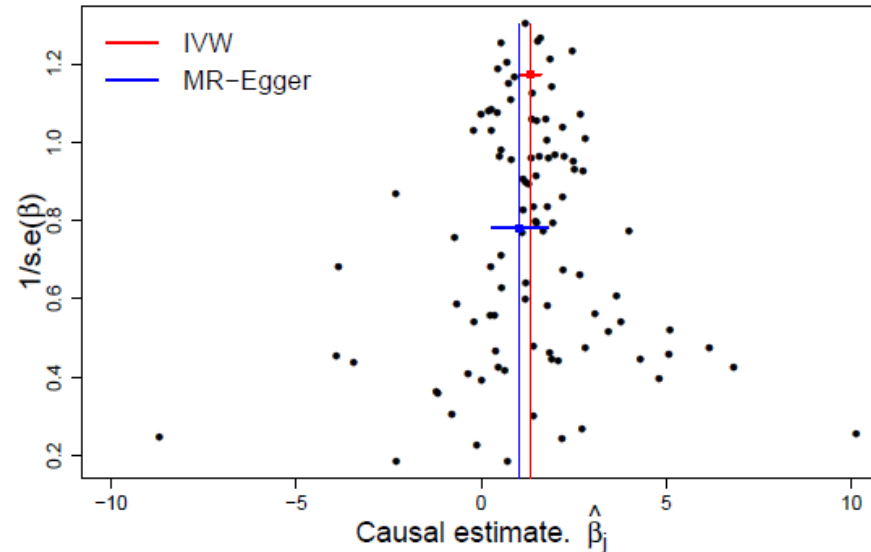
IVW model:
$$\hat{\beta}_{Yk} = \underbrace{\beta_{IVW}}_{\text{Slope}} \hat{\beta}_{Xk} + \varepsilon_{Yk}$$

MR-Egger model:
$$\hat{\beta}_{Yk} = \beta_0 + \underbrace{\beta_{Egger}}_{\text{Slope}} \hat{\beta}_{Xk} + \varepsilon_{Yk}$$

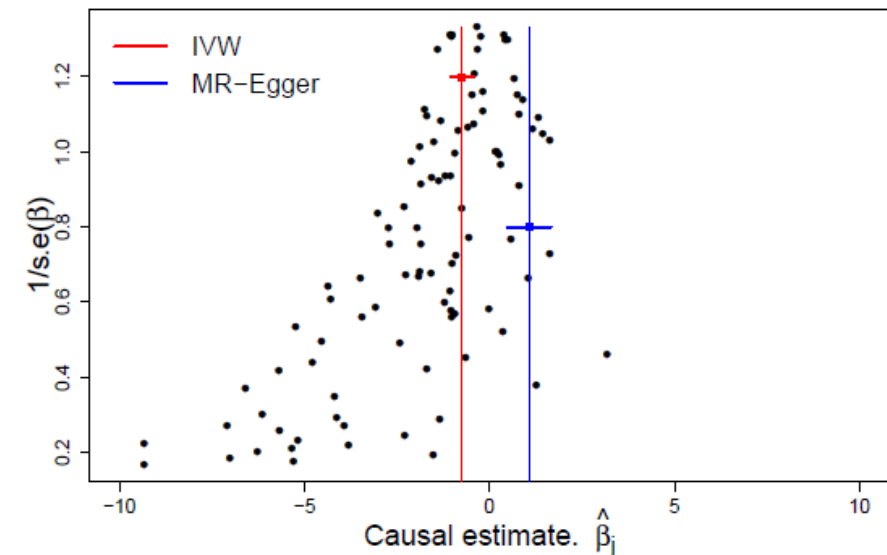
- β_0 is the intercept term. β_0 can be interpreted as the average pleiotropic effect across all genetic variants. A non-zero β_0 indicates directional pleiotropy.
- β_{Egger} is the causal estimate adjusted for directional pleiotropy



Funnel plot: balanced versus directional pleiotropy



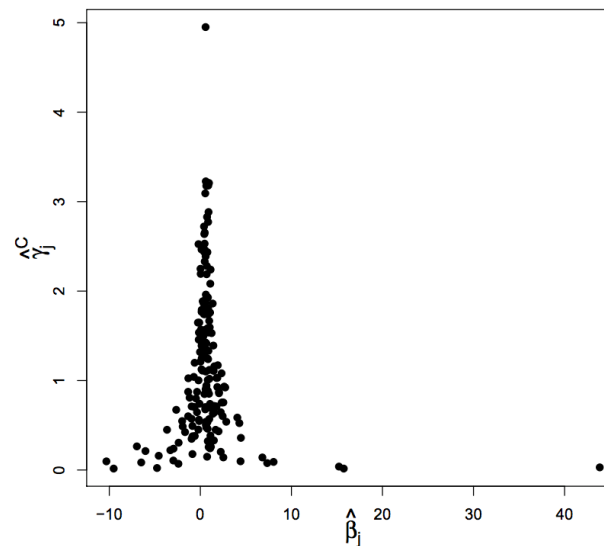
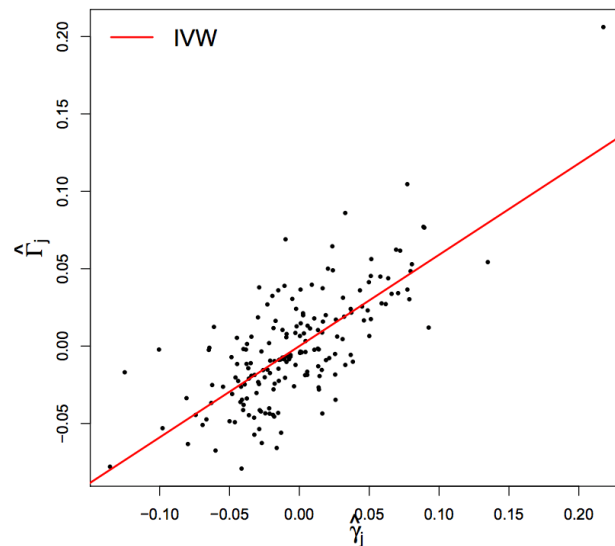
Funnel is symmetric -> pleiotropy appears to be balanced so IVW is okay



Funnel is asymmetric -> pleiotropy appears to be directional so IVW is not okay

MR-Egger regression

Example: height and lung function

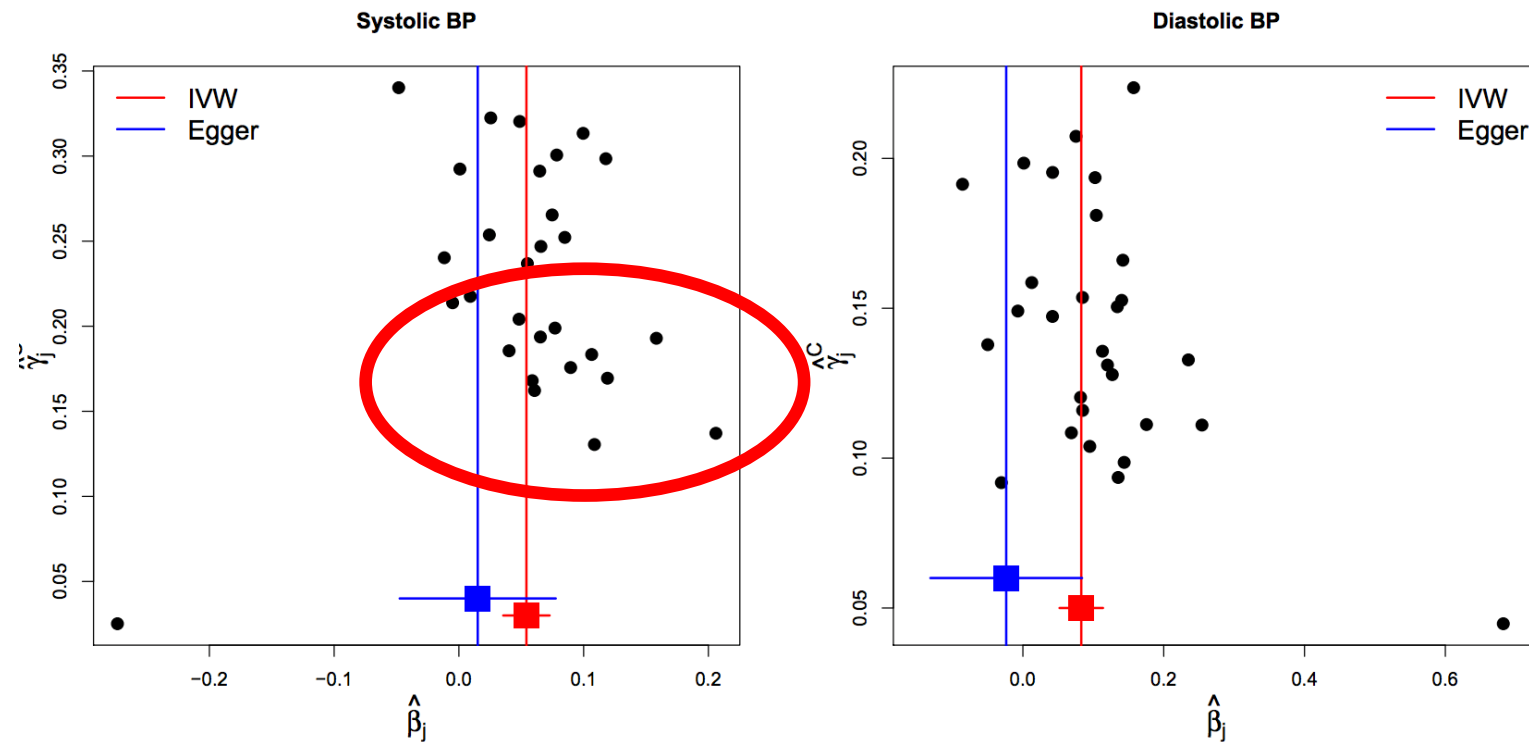


IVW = 0.59 (95% CI: 0.50, 0.67)

MR-Egger = 0.58 (95% CI: 0.50, 0.67);
intercept = -0.001 p=0.5

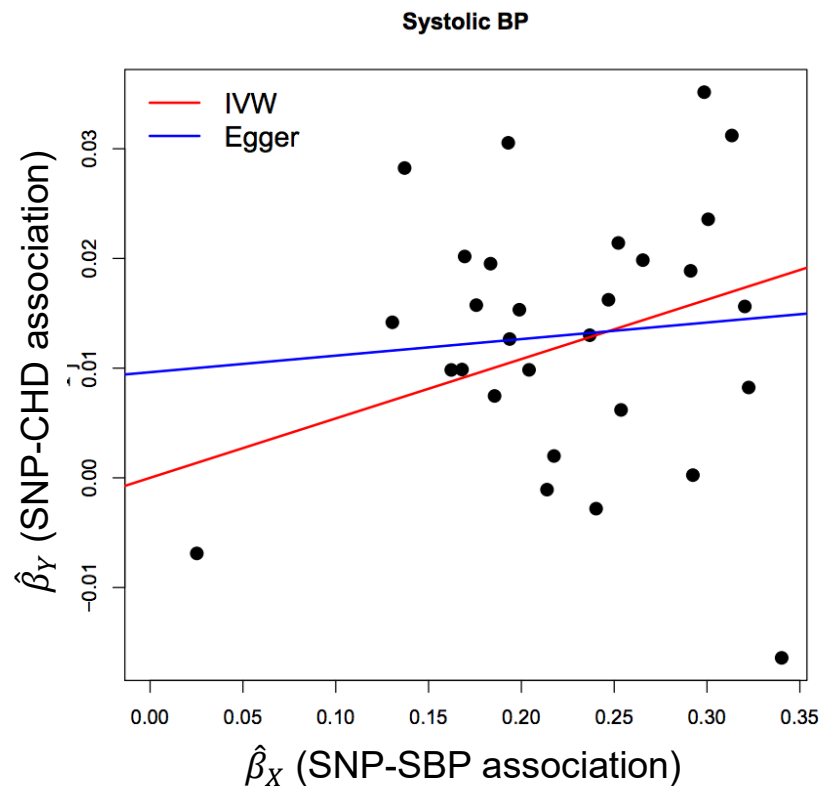
MR-Egger regression

Example: BP and Coronary Heart Disease

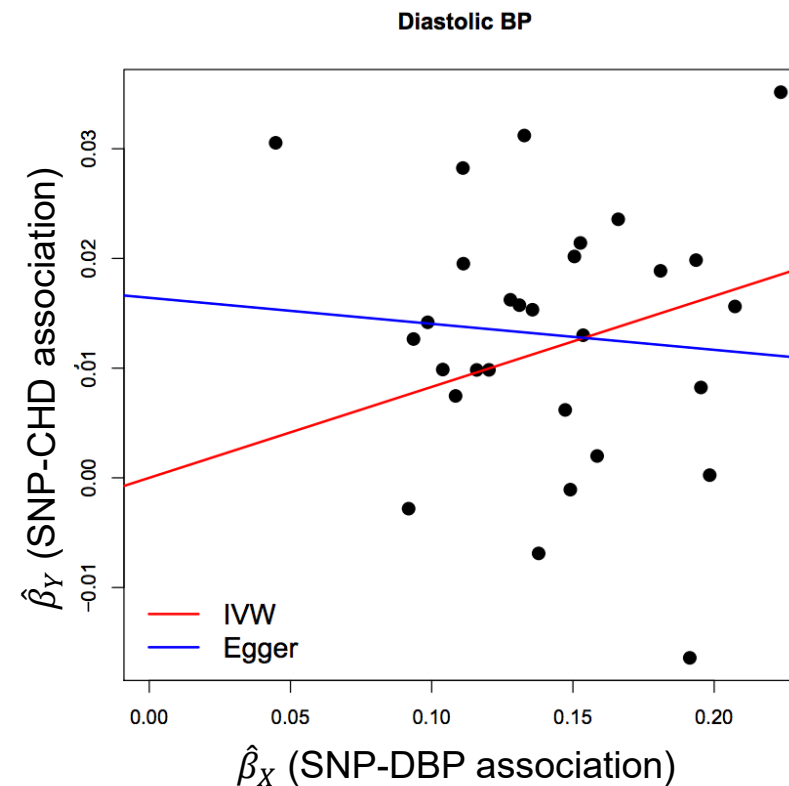


MR-Egger regression

Example: BP and Coronary Heart Disease



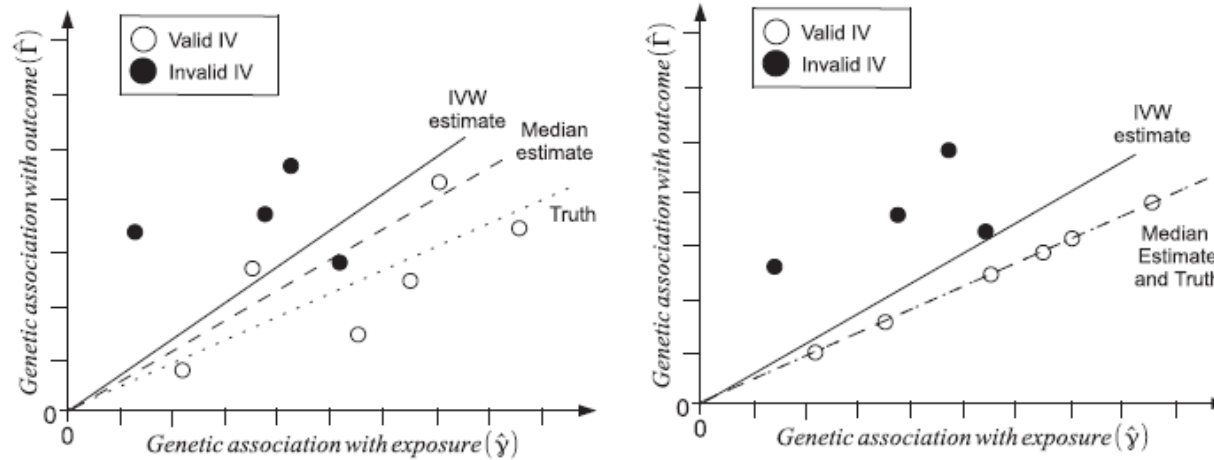
IVW= 0.054 logOR/mmHg $p=4 \times 10^{-6}$
Egger = 0.015 logOR/mmHg $p=0.6$



IVW= 0.083 logOR/mmHg $p=1 \times 10^{-5}$
Egger = -0.024 logOR/mmHg $p=0.7$

Median based methods (Median Estimator)

Order causal estimates (Wald ratio) and take the median.



Assumption: >50% of the instrumental variables are valid.

No restrictions need to be placed on the invalid IVs:

- InSIDE assumption not required
- Violations of #2 and #3 MR assumptions are allowed

Figure 2. Fictional example of a Mendelian randomization analysis with 10 genetic variants—six valid instrumental variables (hollow circles) and four invalid instrumental variables (solid circles) for finite sample size (left) and infinite sample size (right) showing IVW (solid line) and simple median (dashed line) estimates compared with the true causal effect (dotted line). The ratio estimate for each genetic variant is the gradient of the line connecting the relevant datapoint for that variant to the origin; the simple median estimate is the median of these ratio estimates.

Median based methods

Simple median estimation

- Simple median estimator:
 - Odd number of genetic variants: middle ratio estimate
 - Even number of genetic variants: median is interpolated between the two middle estimates

$$\left(\frac{1}{2}(\hat{\beta}_k + \hat{\beta}_{k+1})\right)$$

- Inefficient when the precision of individual variants varies considerably

	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_5$	$\hat{\beta}_6$	$\hat{\beta}_7$	$\hat{\beta}_8$	$\hat{\beta}_9$	$\hat{\beta}_{10}$
Simple median										
Weight ($1/v_k$)	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$
Percentile (p_k)	5	15	25	35	45	55	65	75	85	95
Weighting 1										
Weight ($1/v_k$)	$\frac{1}{30}$	$\frac{2}{30}$	$\frac{3}{30}$	$\frac{4}{30}$	$\frac{5}{30}$	$\frac{5}{30}$	$\frac{4}{30}$	$\frac{3}{30}$	$\frac{2}{30}$	$\frac{1}{30}$
Percentile	1.67	6.67	15.00	26.67	41.67	58.33	73.33	85.00	93.33	98.33
Weighting 2										
Weight ($1/v_k$)	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{10}{36}$	$\frac{8}{36}$	$\frac{5}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$
Percentile (p_k)	2.78	9.72	27.78	52.78	70.83	81.94	88.89	93.06	95.83	98.61

Simple median =

$$\frac{\hat{\beta}_5 + \hat{\beta}_6}{2}$$

Median based methods

Weighted median estimation

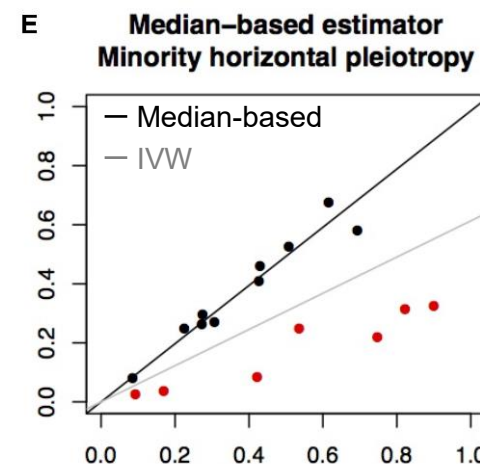
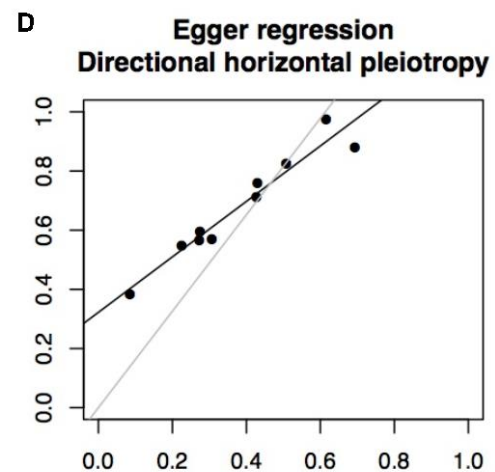
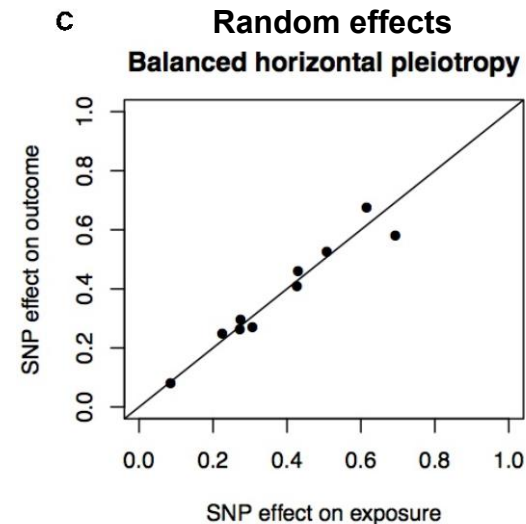
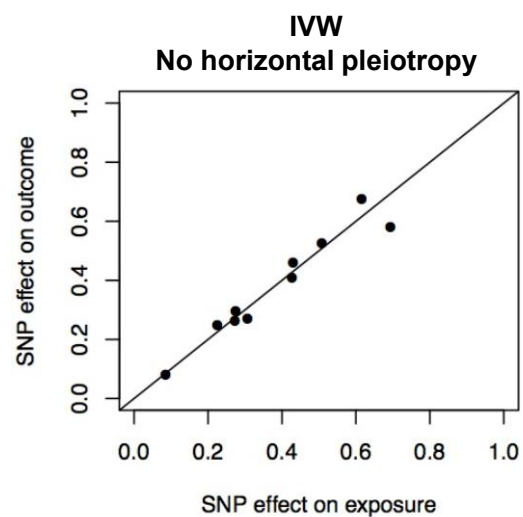
- Weighted median estimator takes into account the differing precisions
- Weighted median: $\hat{\beta}_{WM} = \hat{\beta}_3 + (\hat{\beta}_4 - \hat{\beta}_3) \times \frac{50 - 27.78}{52.78 - 27.78}$
- Suggested weights: inverse of the variance of the ratio estimate: $w'_k = \frac{\hat{\beta}_{Z_kX}^2}{\sigma_{Z_kY}^2}$

	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_5$	$\hat{\beta}_6$	$\hat{\beta}_7$	$\hat{\beta}_8$	$\hat{\beta}_9$	$\hat{\beta}_{10}$
Simple median										
Weight ($1/v_k$)	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$
Percentile (p_k)	5	15	25	35	45	55	65	75	85	95
Weighting 1										
Weight ($1/v_k$)	$\frac{1}{30}$	$\frac{2}{30}$	$\frac{3}{30}$	$\frac{4}{30}$	$\frac{5}{30}$	$\frac{5}{30}$	$\frac{4}{30}$	$\frac{3}{30}$	$\frac{2}{30}$	$\frac{1}{30}$
Percentile	1.67	6.67	15.00	26.67	41.67	58.33	73.33	85.00	93.33	98.33
Weighting 2										
Weight ($1/v_k$)	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{10}{36}$	$\frac{8}{36}$	$\frac{5}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$
Percentile (p_k)	2.78	9.72	27.78	52.78	70.83	81.94	88.89	93.06	95.83	98.61

$$\hat{\beta}_{WM} = \hat{\beta}_5 + (\hat{\beta}_6 - \hat{\beta}_5) \times \frac{50 - 41.67}{58.33 - 41.67}$$

$$\hat{\beta}_{WM} = \hat{\beta}_3 + (\hat{\beta}_4 - \hat{\beta}_3) \times \frac{50 - 27.78}{52.78 - 27.78}$$

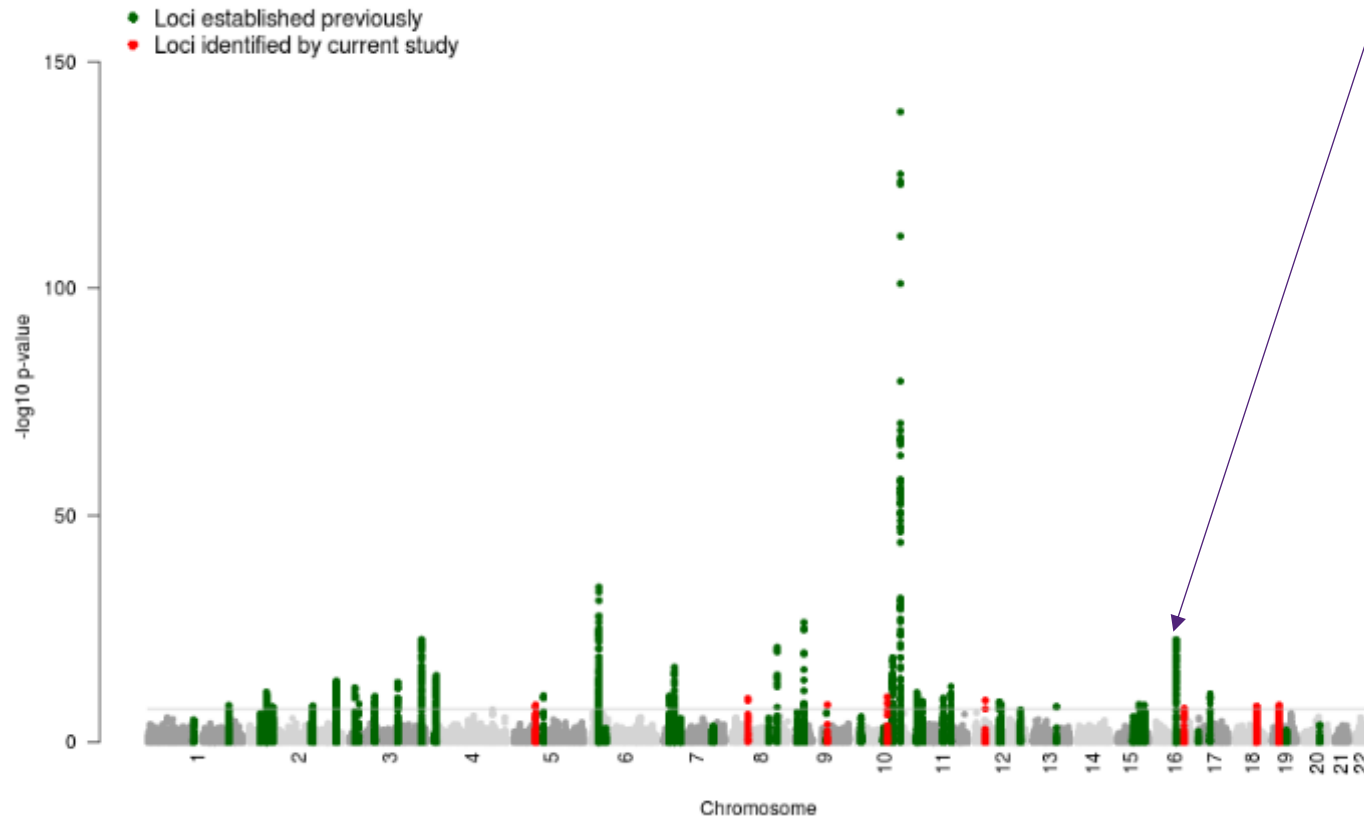
Summary of robust estimators



● SNPs associated with outcome via an independent pathway.

Reverse causal instruments

Problem: MR of type 2 diabetes on BMI



GWAS of T2D reveals *FTO* variant

- Famously associated with BMI
- A reverse causal instrument?

FTO → T2D → BMI

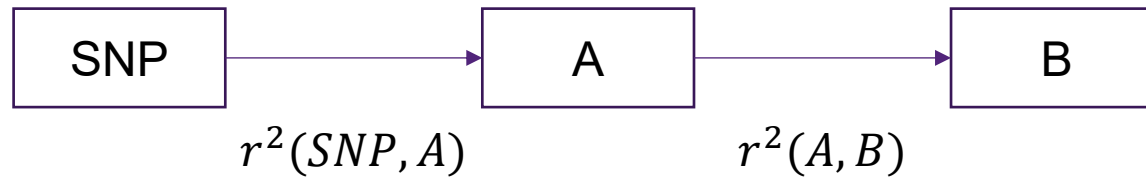
T2D ← BMI ← *FTO*

FTO may be associated with T2D through its effect on BMI
 >> Not a good instrument for T2D

Can we avoid including reverse-causal SNPs as instruments?

Steiger filtering test

- If SNP causes A and A causes B
- The effect of SNP on A should be larger than the effect of SNP on B

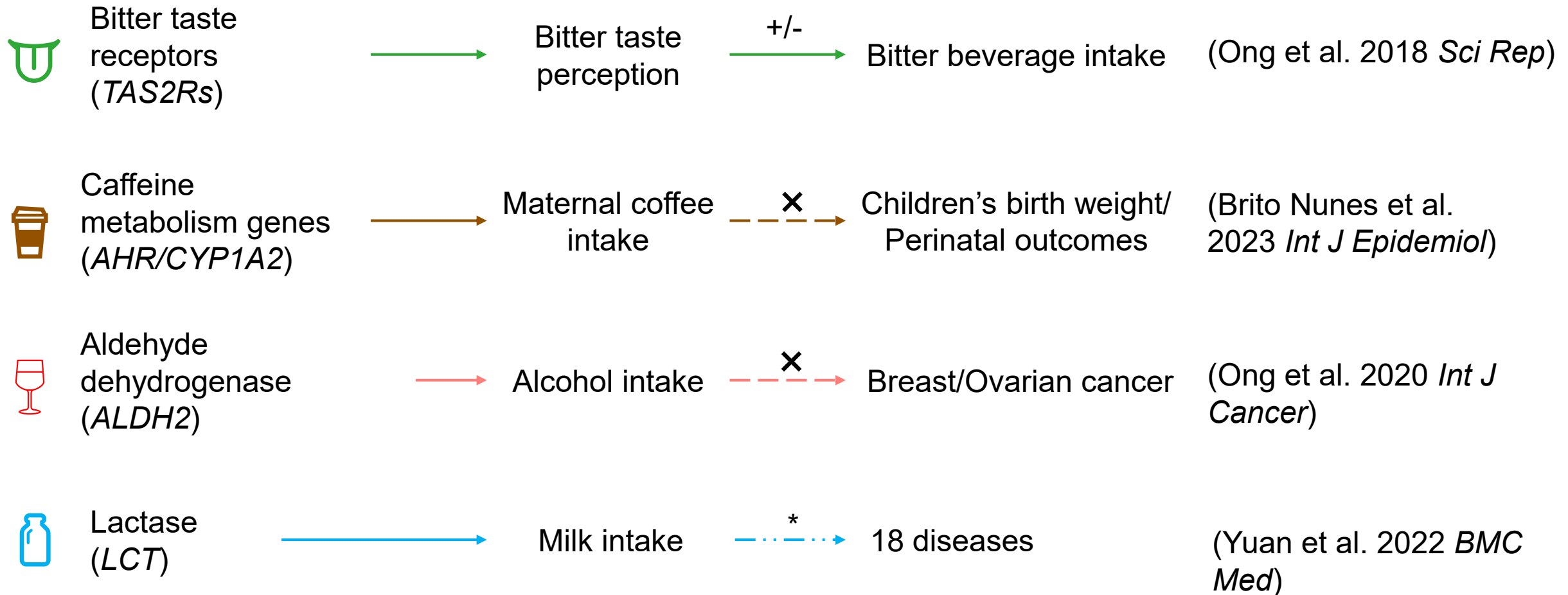


Expect that

$$r^2(SNP, B) = r^2(SNP, A) \times \underbrace{r^2(A, B)}_{\text{This term is } < 1}$$

- Steiger test used to evaluate if $r^2(SNP, A) > r^2(SNP, B)$
- If this is not satisfied, infer that this instrument is not influencing the exposure primarily.

Ideal instruments are genetic variants with a known biological function related to the exposure



TwoSampleMR R Package

TwoSampleMR 0.6.4 Guide ▾ Functions Changelog

Search for

Source

Mendelian randomization with GWAS summary data

A package for performing Mendelian randomization using GWAS summary data. It uses the [IEU GWAS database](#) to obtain data automatically, and a wide range of methods to run the analysis. You can use the [MR-Base web app](#) to try out a limited range of the functionality in this package, but for any serious work we strongly recommend using this R package.

January 2020 major update

We have made substantial changes to the package, database and reference panels. For full details of the changes, please visit <https://mrcieu.github.io/TwoSampleMR/articles/gwas2020.html>

Installation

Users running Windows and macOS, to install the latest version of TwoSampleMR please install from our MRC IEU r-universe

```
install.packages("TwoSampleMR", repos = c("https://mrcieu.r-universe.dev", "https://cloud.r-project.org"))
```

Users running Linux or WebR please see the [following instructions](#).

To update the package run the same command again.

Installing from source

```
install.packages("remotes")
remotes::install_github("MRCIEU/TwoSampleMR")
```

To update the package just run the `remotes::install_github("MRCIEU/TwoSampleMR")` command again.

Docker

A docker image containing R with the TwoSampleMR package pre-installed is available here: <https://hub.docker.com/r/mrcieu/twosamplemr>

Links

[Browse source code](#)
[Report a bug](#)

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Citation

[Citing TwoSampleMR](#)

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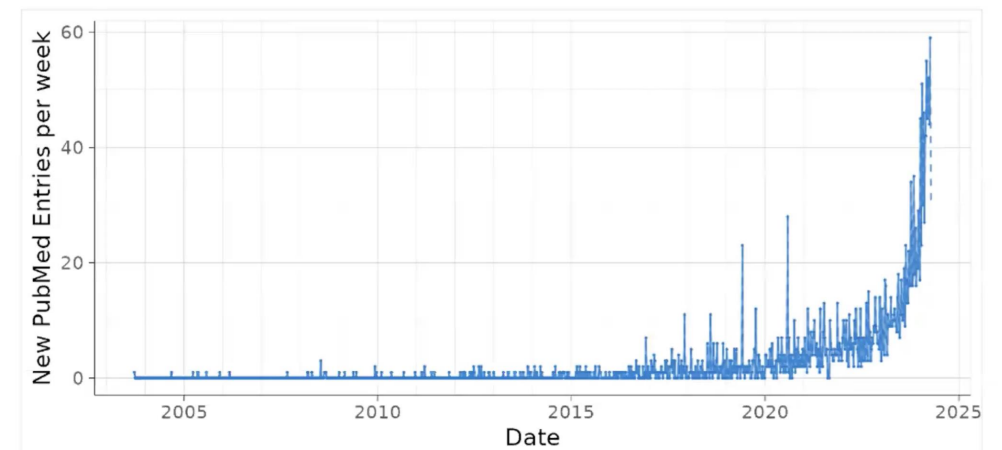
Author 

Tom Palmer

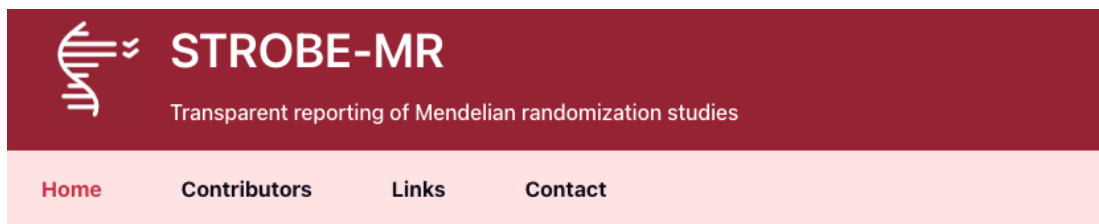
Author 

Dev status

 R-CMD-check passing
 lifecycle experimental

DOI [10.5281/zenodo.10684540](https://doi.org/10.5281/zenodo.10684540)
 codecov 36%
 r-universe 0.6.4


STROBE-MR



Welcome to the STROBE-MR website!

About: STROBE-MR stands for “Strengthening the Reporting of Observational Studies in Epidemiology using Mendelian Randomization”. Inspired by the original STROBE checklist, the STROBE-MR guidelines were developed to assist researchers in reporting their Mendelian randomization studies clearly and transparently. Adopting STROBE-MR should help readers, reviewers, and journal editors evaluate the quality of published MR studies.

The STROBE-MR **checklist** contains 20 items recommended to address in reports of Mendelian randomization studies.

The **Statement** document describes the process of developing the checklist and the complementary Explanation and Elaborations document.

The **Explanation and Elaboration** document explains the items of the STROBE-MR checklist, along with their rationale and examples of transparent reporting.

All documents and publications produced by the STROBE-MR Initiative are open-access and available for download on this website.

MR Dictionary



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The definitive list of terms for Mendelian randomization research















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[Inverse variance weighted \(IVW\)](#)
[fixed effects estimate](#)
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[Cis- and trans-variants](#)
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 Pleiotropy-robust two-sample methods	 Model selection and averaging approaches
 Heterogeneity and outlier detection	 Resources and software

Summary

- MR uses natural randomization to mimic an RCT
- It is useful, data is abundant, but it is not a panacea for causal inference
- Often valuable for proving that a hypothesized association is not causal
- Horizontal pleiotropy is one of the main threats to the validity of MR studies
 - Multiple methods developed to detect and adjust for horizontal pleiotropy
- Crucial to perform sensitivity analyses and obtain metrics regarding the likely reliability of the MR estimates
- Consistency of results across methods is key to reliable causal inference

Additional References

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